

Teaching Algebra in the Primary Classroom: Functions The ‘Function’ Machine

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Introduction

‘Functions’ provide opportunities for children to develop algebraic thinking skills e.g. generalisation. Work with functions facilitates pupils to identify and describe mathematical patterns and relationships using multiple representations e.g. words, tables, symbols. Therefore, consensus exists that the concept of ‘functions’ should take a central place in the teaching of algebra, particularly at the elementary (primary) level (Suh, 2007).

Within the Primary School Mathematics Curriculum, ‘functions’ best fit into the **Algebra** strand unit of **‘Rules and Properties’** and meets the objective *‘The child will be enabled to identify and record verbal and simple symbolic rules of number patterns’*.

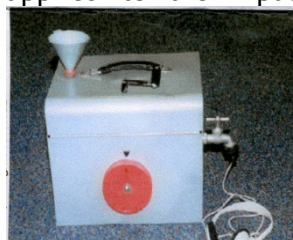
At first the word ‘functions’ may provoke memories of formal symbolic manipulation of abstract equations such as $f(x) = 2x - 3$, which reflect many of our post-primary school experiences (Blanton and Kaput, 2003; Suh, 2007). To teach primary school pupils using such a didactic approach would be potentially disastrous. In the context of primary school, a function should be seen as a *rule*. In order to make the concept accessible to young children, one common approach is the use of a ‘function machine’.

This article explores the use of a function machine as a mechanism for developing understanding of the concept of functions. Subsequently, the article outlines a ‘functions’ lesson suitable for 4th class which was developed by student teachers in Mary Immaculate College who were studying mathematics education as their curriculum specialisation (Sarah Browne, Patrick Jones, John O’ Connor, Stephanie O’ Sullivan and Luis Miquel Moro López). These students were engaged in *Japanese Lesson Study* in conjunction with the authors of this report. The study involved the development of an

initial lesson through a process of research and preparation. They subsequently taught the lesson in two different school settings, an activity which facilitated the introduction of modifications to their lesson plan at each stage of the process.

Teaching: the Function-Machine Concept

In the past, function machines were drawn on the board (highlighting characteristics such as input, output) to facilitate children to understand the concept of functions. However over time, ‘physical models’ of function machines have been promoted as an interesting and enjoyable way to introduce the concept (Reeves, 2005/06). This also has the potential to facilitate conceptual understanding. It is suggested that the machine may be left in the room for a number of days prior to use to heighten pupils’ curiosity and interest. Function machines may be made from old machines such as radios or heaters. It is essential that the function machine has a definite place for the ‘input’ (e.g. funnel), ‘output’ (e.g. earphones) as well as a dial which facilitates the rule to be applied to the input (see picture below).



(Reeves, 2005/06)

In such cases, the children can work towards uncovering the ‘function’ (rule) by examining the various inputs and outputs presented. In such a process the teacher/pupil will pretend to put a number into the machine (through the funnel) and pretend to receive an output (through the ear phones) on rotating the dial clockwise. Efficient mental mathematics is essential for the smooth presentation of this activity. Subsequent activities may include

pupils recording input and outputs in an effort to predict the rule. Once they have made a prediction they can test it by working out subsequent outputs. Much practice can be gained where pupils move from working with simple one-step rules such as 'add 2' to more complex 2-step rules such as 'multiply by 2 and take 1'. In each case, once the 'dial' is reset, a new function/rule is possible. Also if the 'dial' is rotated anti-clockwise, it is possible challenge pupils to work from 'output' to 'input'.

In some cases, function machines facilitate the use of a more concrete experience, where it is possible to see and 'input' materials e.g. interlocking cubes or digit cards as well as the output. For example, one teacher modified an old wardrobe box (see picture below), creating holes for input/output as well as a button that initiated the function/rule. It was possible for a person (teacher/child) to enter the machine to become 'the brains' (accepting inputs and presenting outputs).



(Reeves, 2005/06)

The student teachers in this study adapted this idea and introduced functions through a **physical function machine**. The theme chosen was that of baking in a 'faulty oven'.

'Faulty Oven' Lesson: 4th Class

Introduction to Problem Setting

The following scenario is presented by the teacher:

'Yesterday something very strange happened to me! Well... I love baking and yesterday I made 2 cakes (picture/photo displayed on the white board using projector) and put them into the oven. When I took the cakes out something strange had happened to them'



Invite the children to guess what might have happened.

Expected/Actual Pupil Responses

- You put too much baking powder in
- They were burnt

Present the actual finding

'No, shall I show you? When I opened the oven there seemed to be more than 2 cakes. Look what I found! (picture/photo of 4 cakes displayed by the projector)...Lets count them'

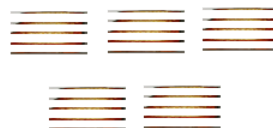


Ask children to predict the reasons for the change.

Expected/Actual Pupil Response

- It added 2
- It doubled them
- It's like a cloning machine
- That must be a special oven

The pupils are subsequently given further opportunities to experience the function/rule that the oven is using through other examples: *'Later yesterday evening I made 5 lasagnes (displayed on the board) because I was having friends over for dinner, but when they were cooked I opened the oven and guess what I found?'*

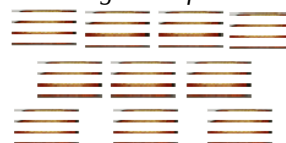


Expected/Actual Pupil Responses

- More lasagnes
- 7 lasagnes
- 10 lasagnes

Prompt/probe where necessary e.g. 'Any other guesses?'; 'Did more or less come out do you think?'

The teacher presents the findings and encourages pupils to identify the rule/pattern which determines the number of items e.g. *'I found 10 lasagnes! I wonder what is wrong with my oven. What is it doing to the food each time?'*



Prompt/probe if necessary e.g. ‘Did you notice anything special happening to the food that I put in each time?’; ‘Do you notice a pattern?’

Expected/Actual Pupil Responses

- I wish I had an oven like that!
- It is making the number in the oven bigger each time
- It is doubling

Pupils’ understanding of the nature of the function/rule can subsequently be tested through teacher questions such as:

‘If I were to bake 3 apple tarts in the oven I wonder how many would be there when I would go to take them out?’



Pupils are introduced to the concept of a function/rule machine:

‘I soon realised that my oven isn’t an oven at all...It’s a magic machine called the ‘Maths Rule Generator’. Do you want to see it?’

A physical ‘Maths Rule Generator’ or ‘Broken Oven’ is presented to the class.

Note: The actual ‘oven’ was made by one of the student teachers using a box, tinfoil, etc.



Lesson Development

Step 1: Presentation of New Language

The language of input, output and rule require explicit introduction. This may be carried out through reference to the ‘Maths Rule

Generator’ e.g. ‘Do you see this door here where we put in the items?’ (point at the door)

Well..the name we give to everything we put **in** the machine is called the **input**’



Ask the children if they could think of anything besides food that they would like to put into the ‘broken oven’ if the oven would double it.

Expected/Actual Pupil Responses

- Money
- Sweets
- Dog
- Pizza

Subsequently, children may be asked to predict the name of things that come out of the ‘Maths rule Generator’ i.e. output



Step 2: Exploring the relationship between input, output and functions/rules

The class is asked if they would like to try the machine. Their attention is focused to the fact that there are different coloured buttons on the ‘Maths Rule Generator’ and advised that each button has a different rule. They are encouraged to predict what the rules might be.

Expected/Actual Pupil Responses

- Add
- Subtract
- Multiply
- Divide

Appropriate prompts include ‘Yesterday when I was baking the lasagnes, I pressed the orange button’

The teacher leads the class in trying to figure out the rule for each of the buttons. Children are invited to the top of the class to oversee the process which consists of inputting materials (counting the quantity), pressing the appropriate button and counting the output.

Note: The class is advised that as food would be too messy, base ten blocks (or interlocking cubes in bundles of 10) will be used instead. These materials are appropriate in size and can be pre-prepared (in cartons) to ensure the smooth flow of the activity.

Throughout the lesson, the input, output, and rule (both guess and actual) are recorded on an activity sheet. Each button is included e.g.



Input	Output	Rule

The teacher may present an enlarged activity sheet to demonstrate the process for the first button. In each case the children will be encouraged to predict the rule by looking at the relationship between the input and the output. As a new pair of values (input and output) are presented, such predictions can be tested (confirmed or rejected).

The rules for the various buttons can vary but may focus initially on addition and multiplication as there is no limitation on the input selected e.g. orange ($\times 2$ or double), blue ($+15$), green ($\times 9$).

In order to facilitate challenge, the final button (yellow in this case) may be a two-step rule or functions ($\times 2 + 3$). Children may be encouraged to predict what a two-step rule would consist of. On presenting a number of related inputs and outputs, pupils are given time to work out the rule/function. In order to promote success, prompts may be given regarding the first step.

Step 3: Independent Group Work

Pupils are given the opportunity to work in groups to generate and identify rules/functions. Following teacher demonstration, pupils will take turns acting as the 'rule generator'. This involves pupils in the group providing inputs (between 0 and 10) and the 'rule generator' working out and sharing the output (using a secret rule e.g. $+5$). Rules for turn-taking may be provided (e.g. clockwise). As in the previous step, the information is recorded facilitating prediction and checking of the rule (see activity sheet which follows)

Generator 2		
Input	Output	Rule
		Guess:
		Real:

Pupils may be encouraged to create a two-step rule if they wish. A hundred square or calculator may be given to each child to support them in their calculations.

Lesson Conclusion

Each group will select a 'rule' to share with the class. One member will act as the 'rule generator' and the remainder of the groups will be challenged to find the rule. During this process, consider providing the class with an output and challenging them to work out the input.

Revise the main points of the lesson e.g. the part of the machine, the types of rules etc. This may be done by asking the children to share their experiences regarding the working of the rule generator e.g. *the rule must always be the same*.

Note: This lesson may be modified to meet the needs of your class group. The 'function machine' may also be used to promote understanding of operations and tables.

References

- Blanton M.L. and Kaput J.J., 2003. Developing Elementary Teachers' "Algebra Eyes and Ears", *Teaching Children Mathematics*, October: 70-77.
- Reeves, C.A., 2005/06. Putting the Fun into Functions, *Teaching Children Mathematics*, December: 250-259.
- Suh, J.M., 2007. Developing "Algebra-Rithmetic" in the Elementary Grades, *Teaching Children Mathematics*, November: 246-253.

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