Calculus

Teacher CPD #6: Related Rates of Change

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EPI·STEM

What are related rates of change?

- As the name suggests, related rates of change are when we are looking to find one rate of change which is related to other rates of change which we are given or can work out.

- Interestingly, related rates of change are simply an application of the chain rule but they are very rarely introduced as such.

- We will use a real-life example to show how this could be introduced.

An oil rig, similar to the one we were looking to build at Rosslare, is located close to Florida in the United States. Recently it sprung a leak in calm seas and the oil spread in a circular patch around the rig. If the radius of the oil patch increases at a rate of 30 metres/hour find an expression for the rate of change of the area of the oil patch over time.

What rate of change are we looking for?

What rate of change are we given?

We want to find $\frac{dA}{dt}$ and we know $\frac{dr}{dt}$ = 30.

How can we relate these two?

$\frac{dA}{dt} = \frac{dr}{dt} \times ??$

$$\frac{dA}{dt} = \frac{dr}{dt} \times \frac{dA}{dr} = 30 \times \frac{dA}{dr}$$

In order to find the derivative $\frac{dA}{dr}$ we need to have a function that relates the area of a circle to its radius.

Can you think of any function we could use?

The area of a circle is given by $A = \pi r^2$ and hence we have a function which provides us with a relationship between Area and Radius

$$\frac{dA}{dr} = 2\pi r$$

$\frac{dA}{dt}$ = 30 $\times 2\pi r$ = 60πr m^2/hour

Another Problem to Solve: Olympic Pools

Rio de Janeiro hosted the 2016 Summer Olympics. The swimming pool constructed at Estádio Olímpico de Esportes Aquáticos was used for all major swimming events at the 2016 Olympics. The width of this pool is half the length of the pool while the depth of the pool is 0.12 times the length.

The pump installed to fill this swimming pool on a daily basis states that it pumps water at a rate of 86 US gallons per metre (GPM). Given that 1 GPM equates to 0.0037854 cubic metres per minute (m3/min), find the rate of change of the depth of the water, whilst the pool is being filled, if the depth of the pool is 2metres.

What we want to find out

Rio de Janeiro hosted the 2016 Summer Olympics. The swimming pool constructed at Estádio Olímpico de Esportes Aquáticos was used for all major swimming events at the 2016 Olympics. The width of this pool is half the length of the pool while the depth of the pool is 0.12 times the length.

What do we know

The pump installed to fill this swimming pool on a daily basis states that it pumps water at a rate of 86 US gallons per minute (GPM). Given that 1 GPM equates to 0.0037854 cubic metres per minute (m^3/min), find the rate of change of the depth of the water, whilst the pool is being filled, if the total depth of the pool is 2 metres.

Possible Solution

Let's go about finding it out.