University of Limerick
Ollscoil Luimnigh

EPI·STEM

Calculus

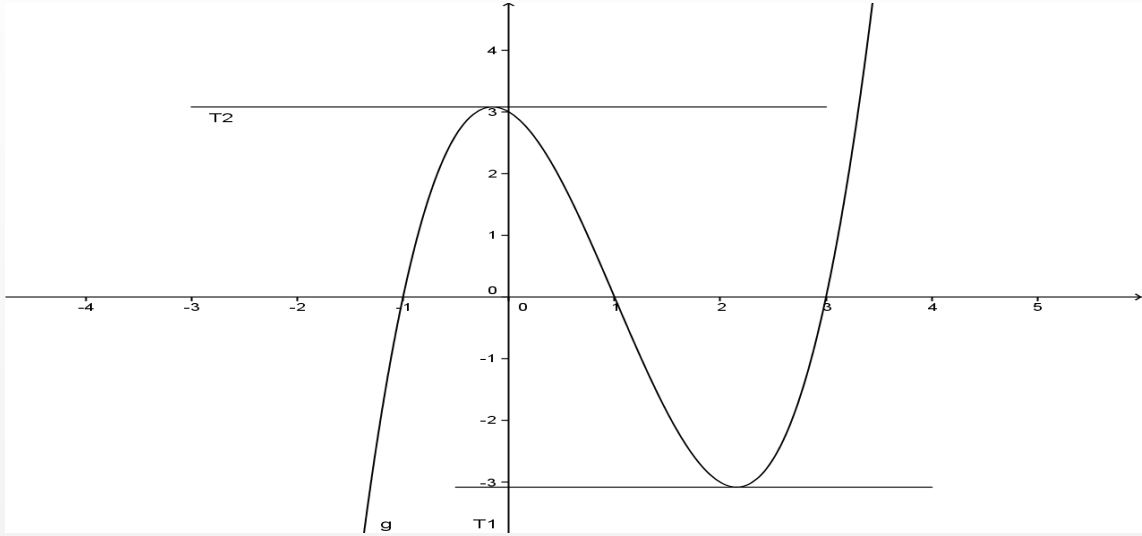
Teacher CPD #5: Maximum and Minimum Points

Potential Student Question

Why do the maximum and minimum points occur at $\frac{dy}{dx} = 0$?

Explanation

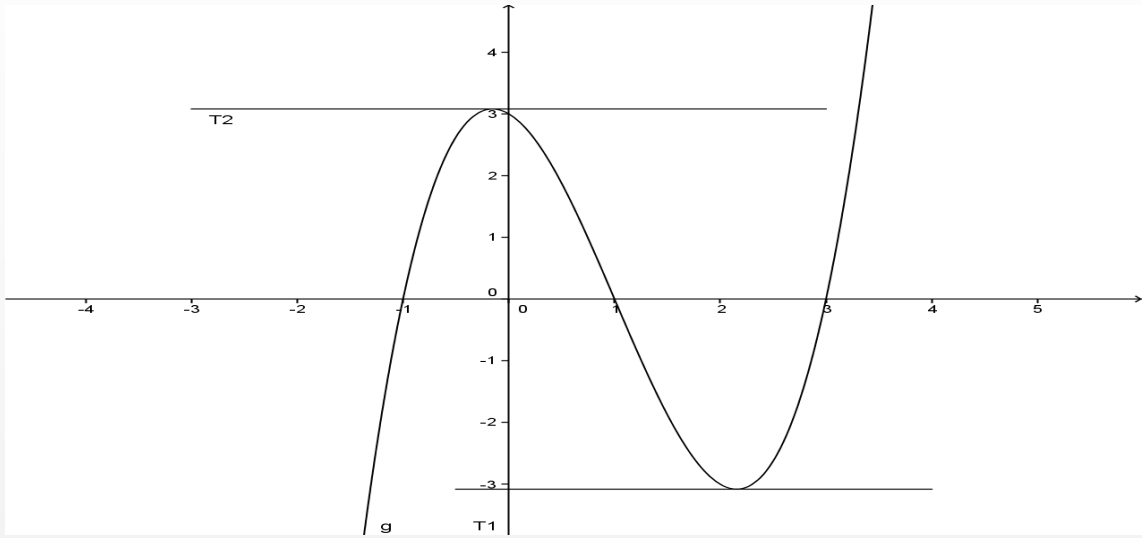
- In a previous video we have discussed the fact that the slope of a tangent to a curve at a point x is equal to $\frac{dy}{dx}$ at the point x.
- When we reach our maximum and minimum turning points the tangents are parallel to the x axis.
- From studying co-ordinate geometry we know that parallel lines have equal slopes so once we know the slope of the x axis we can find the slope of the tangents at the maximum and minimum points.
- What is the slope of the x axis?



Explanation

- The slope of the line $y = 0$ (i.e. the x axis) is zero and hence we can conclude that the slopes of the tangents at the maximum and minimum points are zero.

- Since the slope of a tangent at a given point is equal to the first derivative at that point we can say that at the maximum and minimum points $\frac{dy}{dx} = 0$.



Potential Student Question

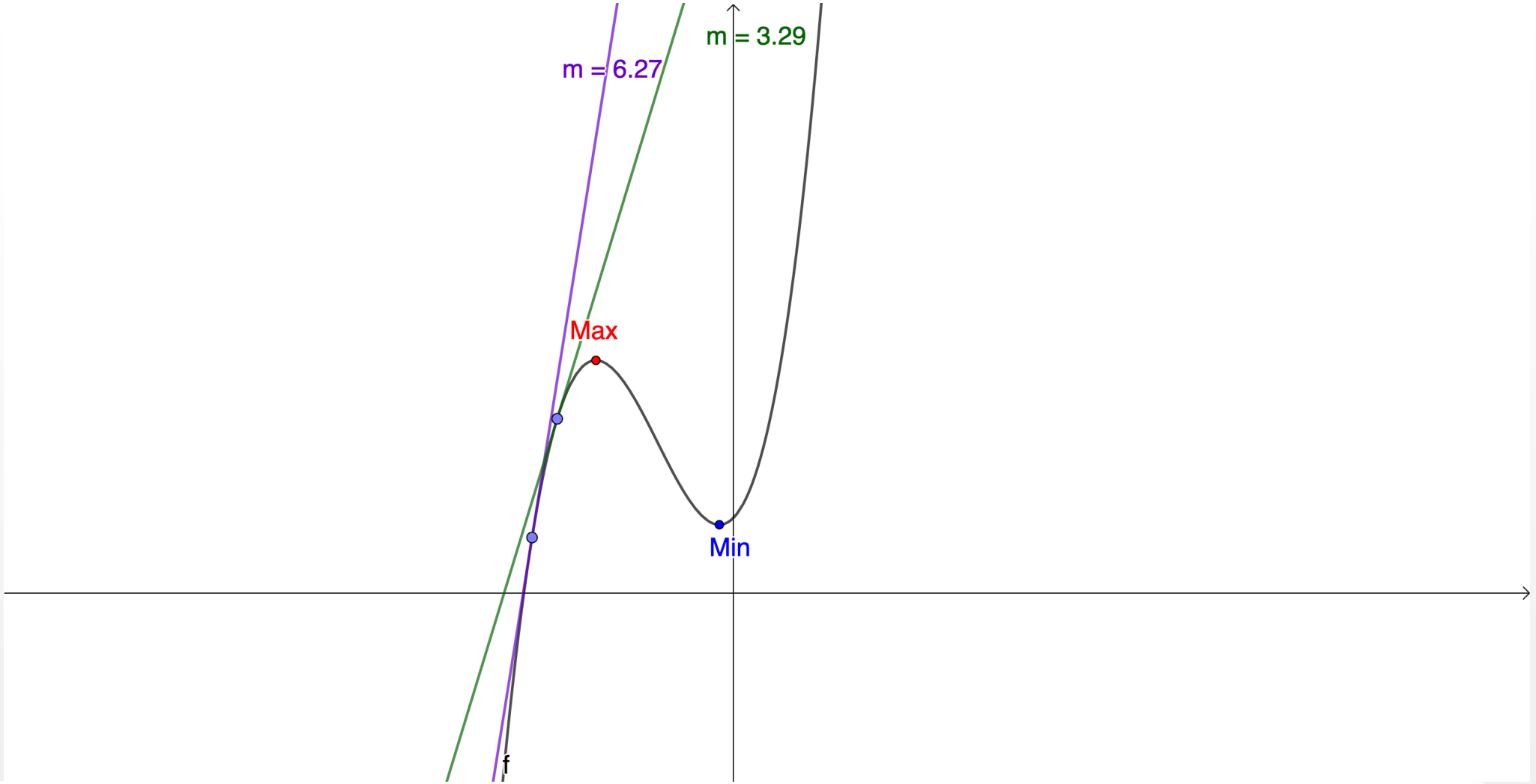
Why does the second derivative test work?

Background Information

- The second derivative test states that if a point (x, y) is a maximum point then at x $\frac{d^2y}{dx^2} < 0$. On the other hand, if a point (t, r) is a minimum point then at t $\frac{d^2r}{dt^2} > 0$.

- Here we will explore why this is the case.

Possible Explanation



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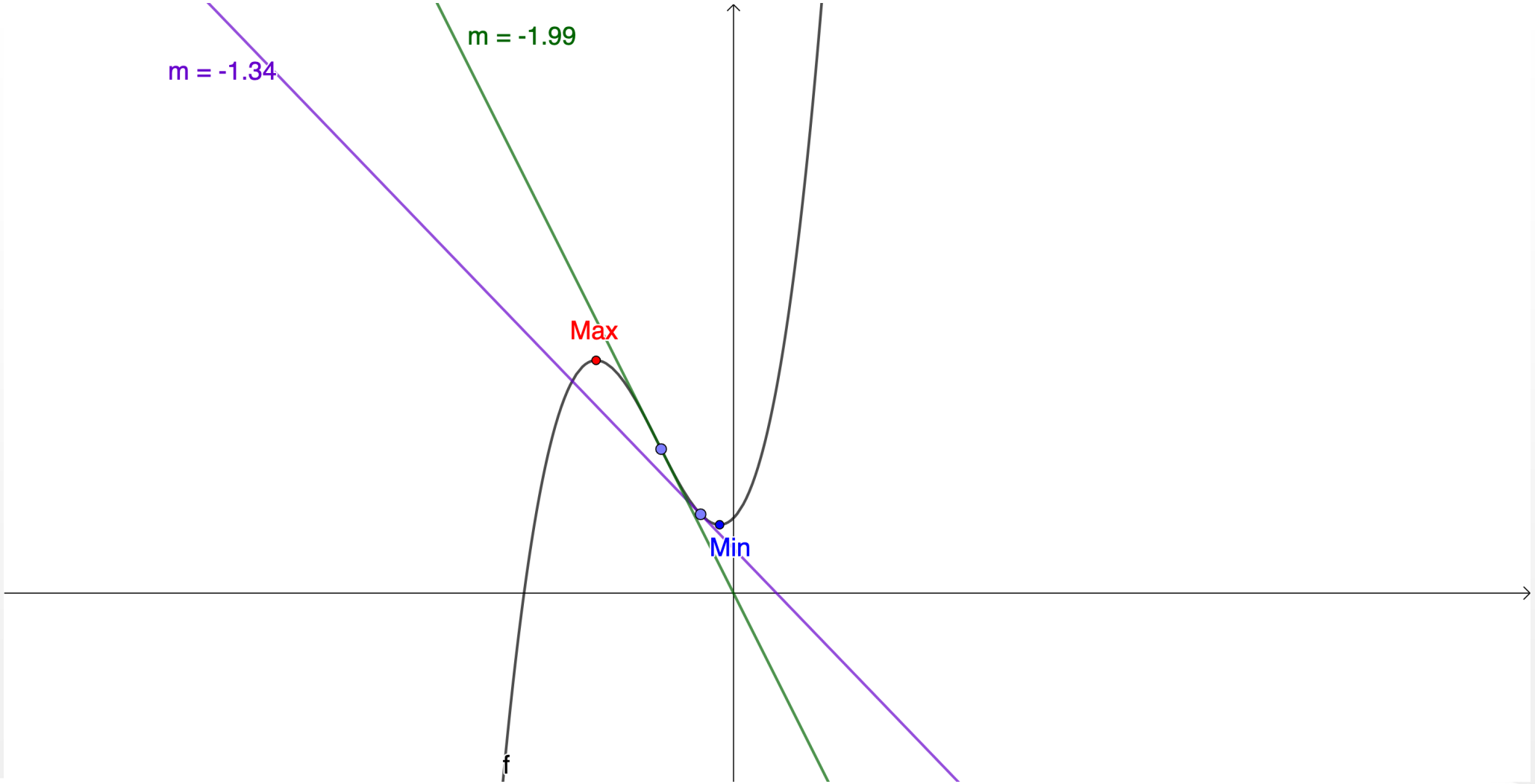

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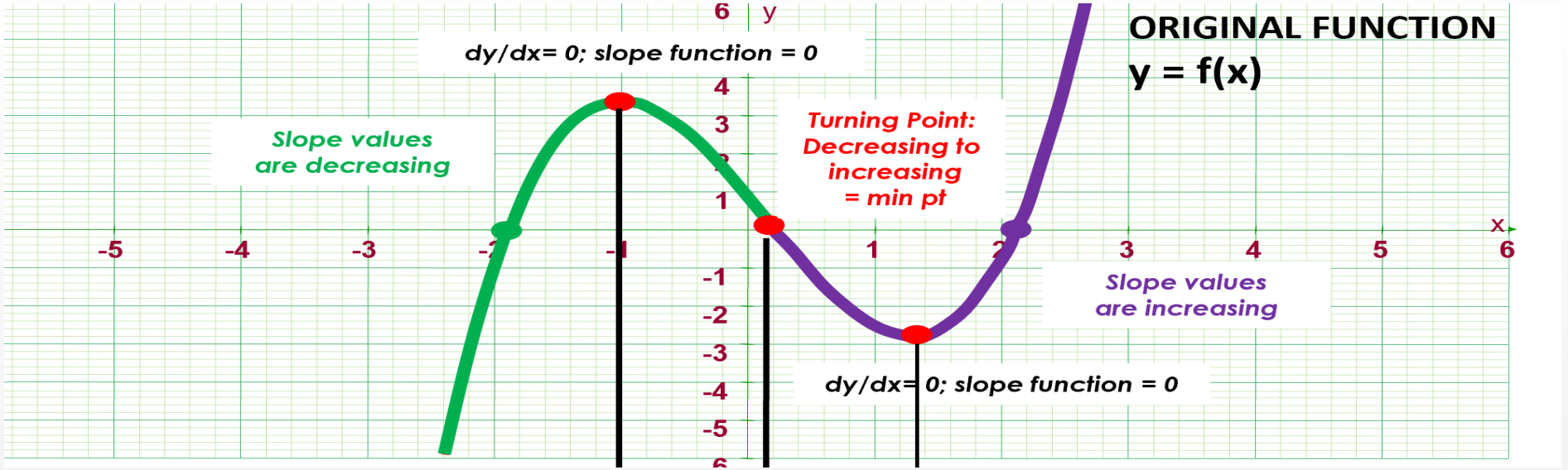
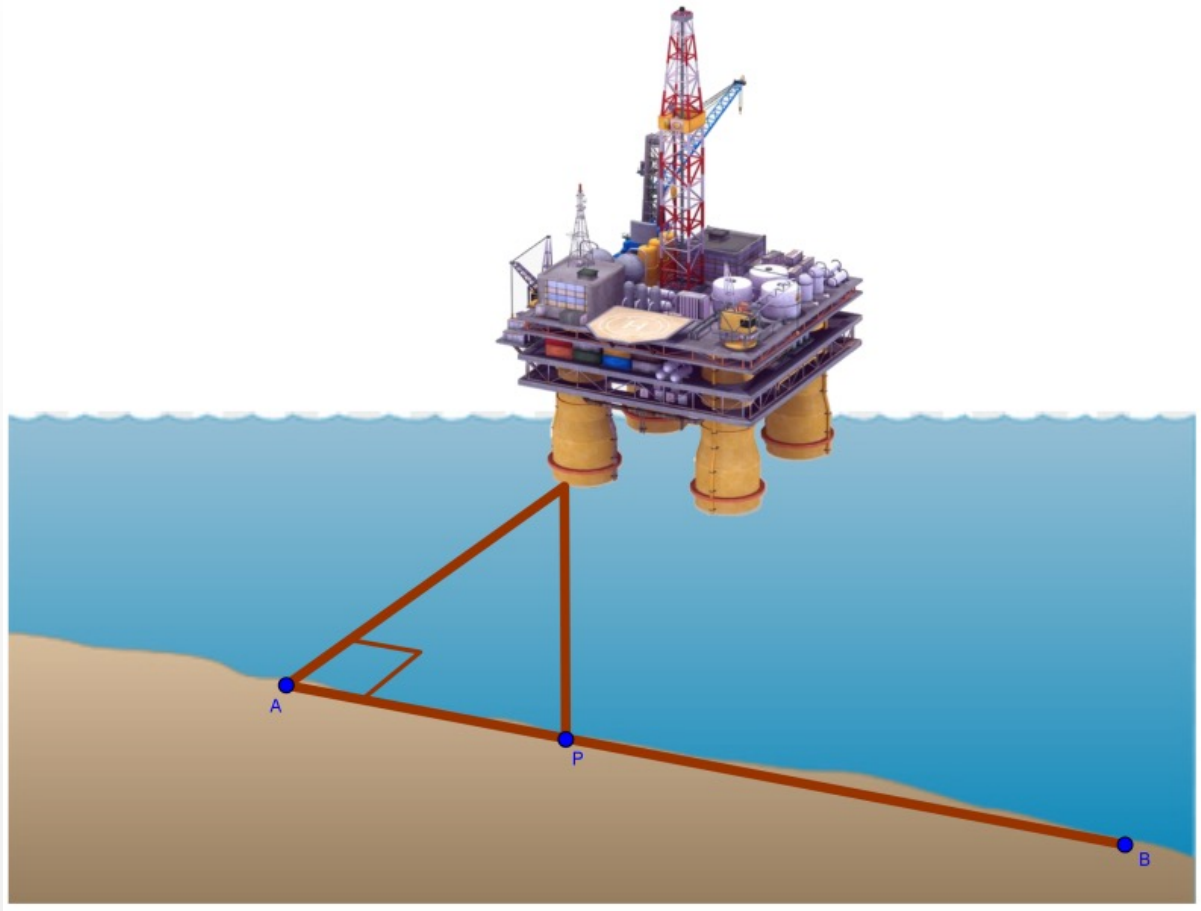


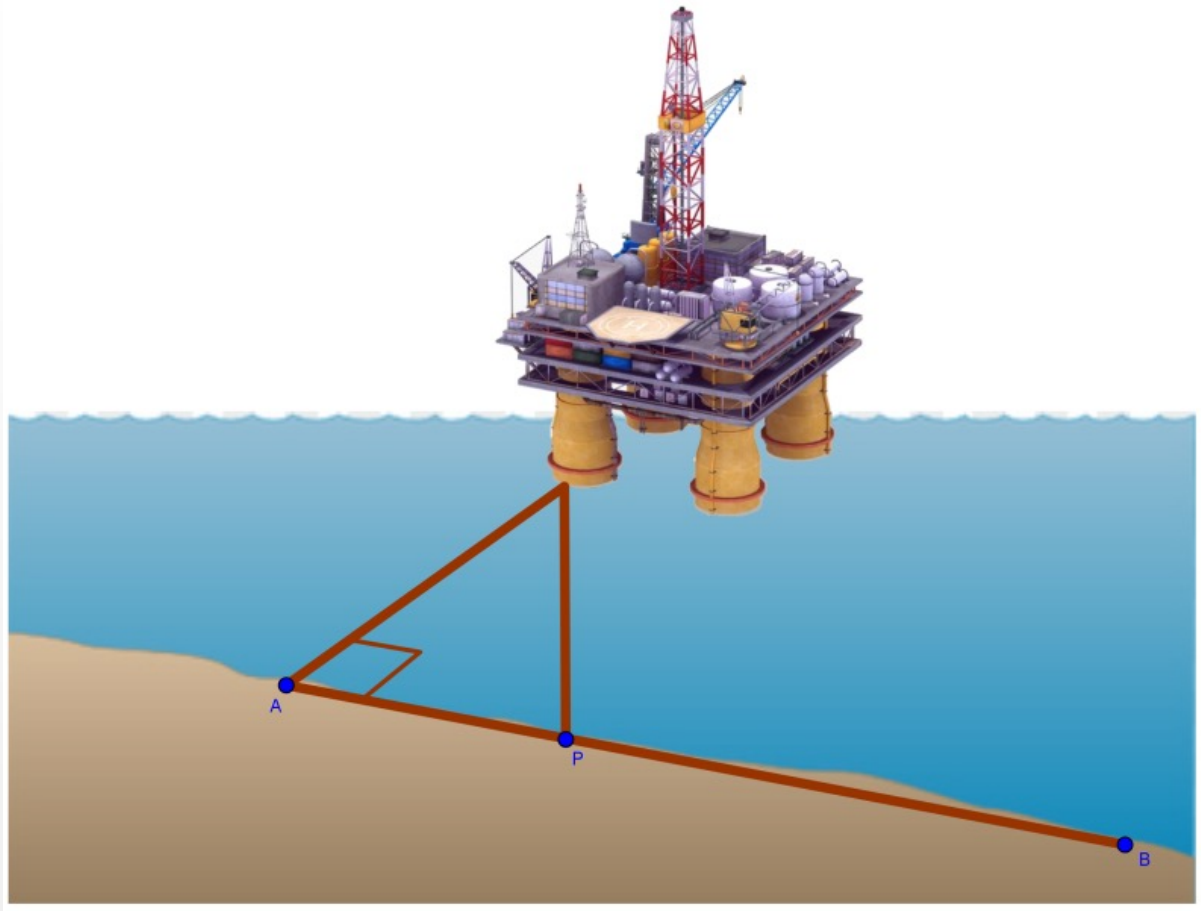
Real Life Application

Plans are in place to locate an offshore oil rig in an area 5 kilometres (km) from the closest onshore point in Rosslare, County Wexford (A) on a straight shoreline, as shown in the diagram. Oil is to be pumped from the rig to another area on Shore Road in Rosslare (B) that is 8km from A by piping it on a straight line under water from the rig to some point on land, P, between A and B and then onto B via a pipeline along the shoreline. The cost of laying pipe is €1,000,000 per km under water and €500,000 per km on land. You are tasked with determining where the underwater pipe should meet the shoreline to minimise costs (i.e. find the optimal location of P).



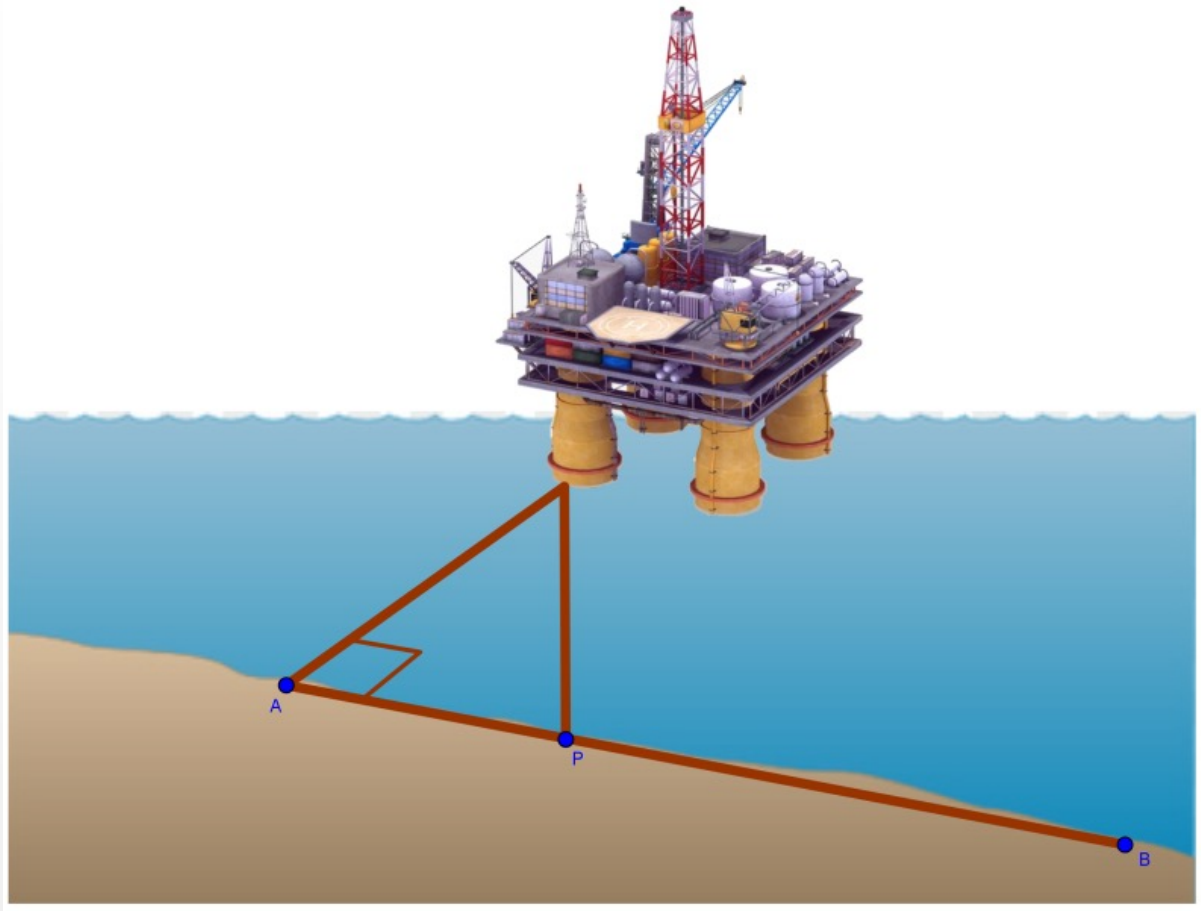
Guided Discovery Approach

Task 1: Find an expression for the distance from the oil rig to the point where the underwater pipe meets the shore i.e. P.



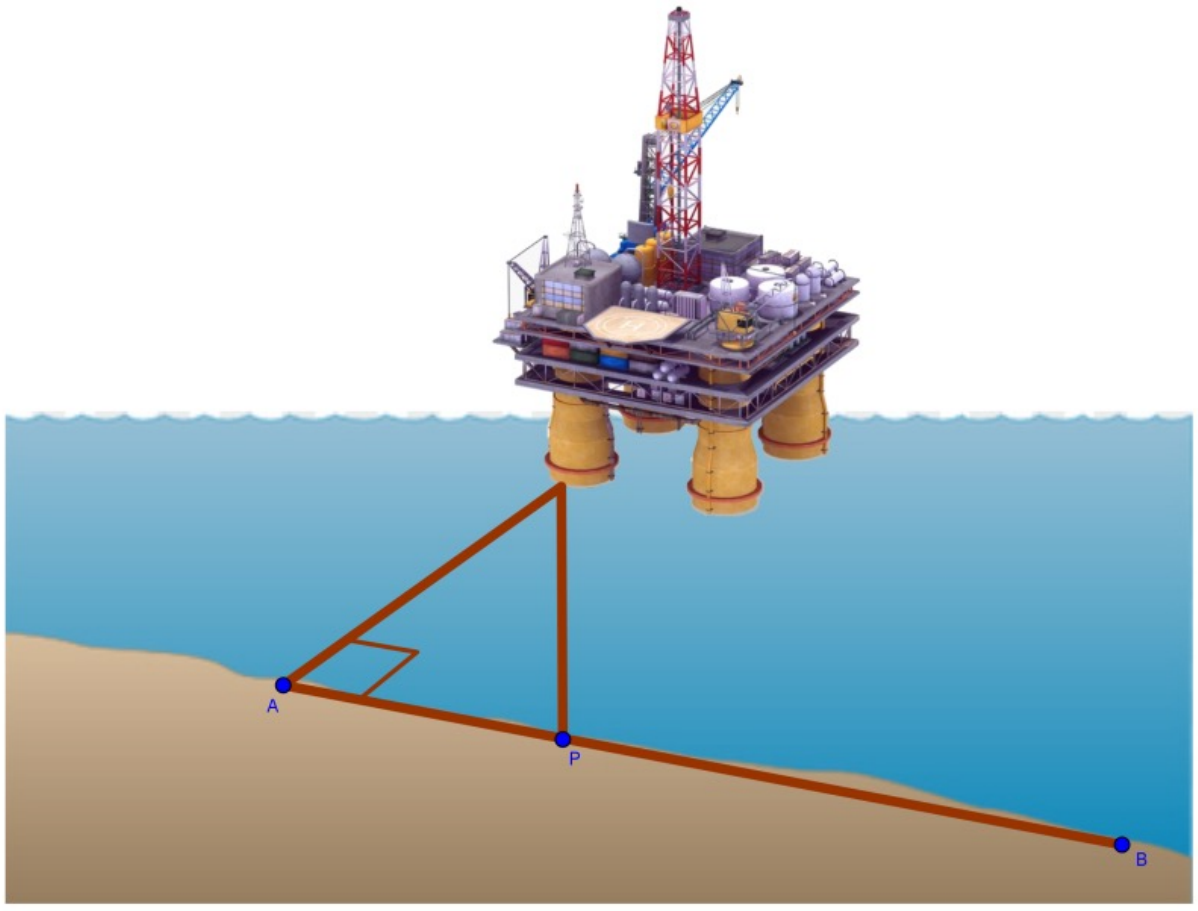
Guided Discovery Approach

Task 2: Find an expression for the length of the pipeline that will run along the shore



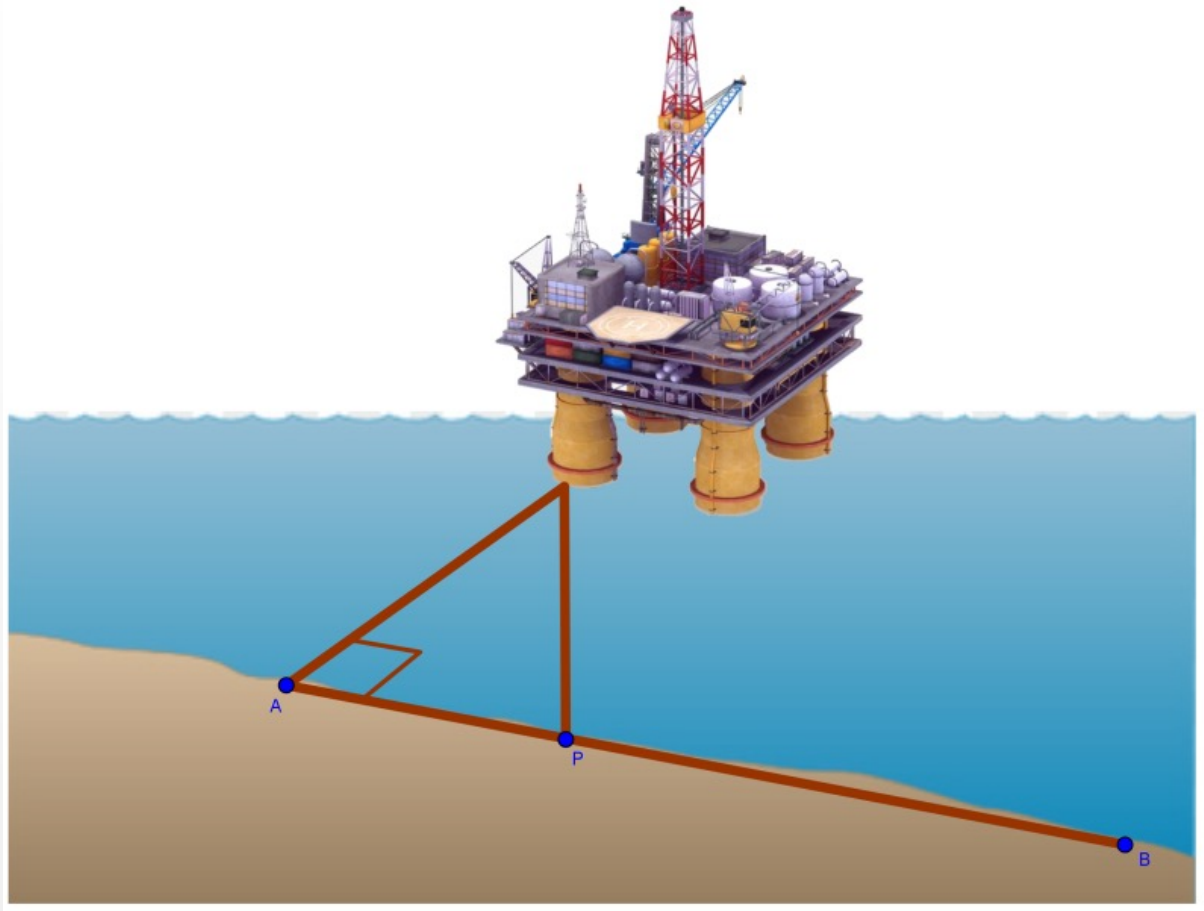
Guided Discovery Approach

Task 3: Find an expression for the cost of the pipeline needed to connect the oil rig with Shore Road and the cost of the pipe running over ground from P to B.



Guided Discovery Approach

Task 4: Now deduce how far P should be located from A in order to minimise the costs.



Some Follow-On/Extension Questions

- How much underwater piping is required?
- What is the minimum cost of the project?
- If the underwater piping was sourced from a new company for 5% less but the overground piping cost 8% more, where should P be located? Would it be better value to go with this new company?

For You To Try

A new Decathalon store is set to open in Limerick in 2023. The manager has been given €1,500 to develop a display area in the new Limerick store. His idea is to build a 600 square metre (m^2) rectangular enclosure in the store's sheltered car park. Three sides of the enclosure will be built of redwood fencing at a cost of €14 per metre while the fourth side will be constructed from a solid cement block at a cost of €28 per metre. Before he reports his idea to the head office he wants to make sure it is a feasible project. If the manager minimises costs, will the money allocated be enough to fund his proposal? The following questions may guide you.

(a) Given that the length of the enclosure is denoted by a metres and its width is denoted by b metres, find an expression for the cost of the project in terms of a and b.

(b) Using the information provided, express a in terms of b. Hence, express the cost of the project in terms of b only.

(c) Find the dimensions of the enclosure that will minimise cost.

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