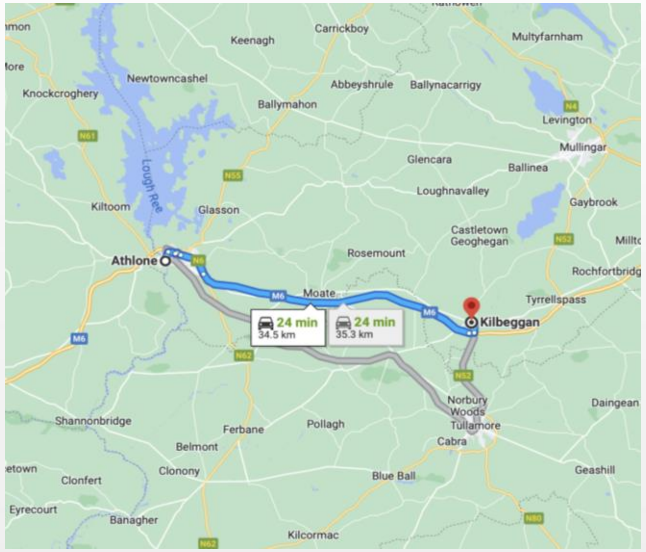
Calculus

Teacher CPD #1: Differentiation from First Principles

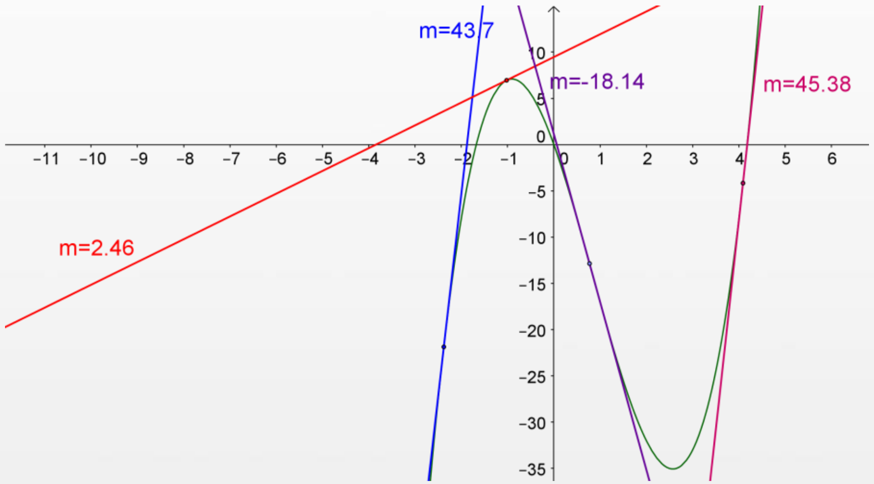
Understanding Calculus

- Calculus is the study of change.
- It is the branch of mathematics that allows us to determine the rate of change of one quantity as another quantity changes.
- For example, as a car travels between two towns, let's say Athlone and Kilbeggan, its distance from the starting point, i.e. Athlone , is constantly changing as time elapses/changes. This rate of change is commonly referred to as speed since speed is calculated by dividing the change in distance by the change in time.
- Can you think of any other rates of change? Are there any covered on the Junior Cycle mathematics curriculum?



Understanding Calculus

- Another example of a rate of change encountered previously would have been when evaluating the slope of a line.
- The slope of a straight line is calculated by finding the change in y values (i.e. $y_2 - y_1$) with respect to the corresponding change in *x* values (i.e. $x_2 - x_1$).
- At Junior Cycle we were able to find the slopes of straight lines using basic co-ordinate geometry since the slope of the line never changed . This is <u>not</u> the case with curves.
- The slope of a curve changes at different points. We can approximate this slope at a certain point by calculating the slope of the tangent to the curve at that point.



Differentiation

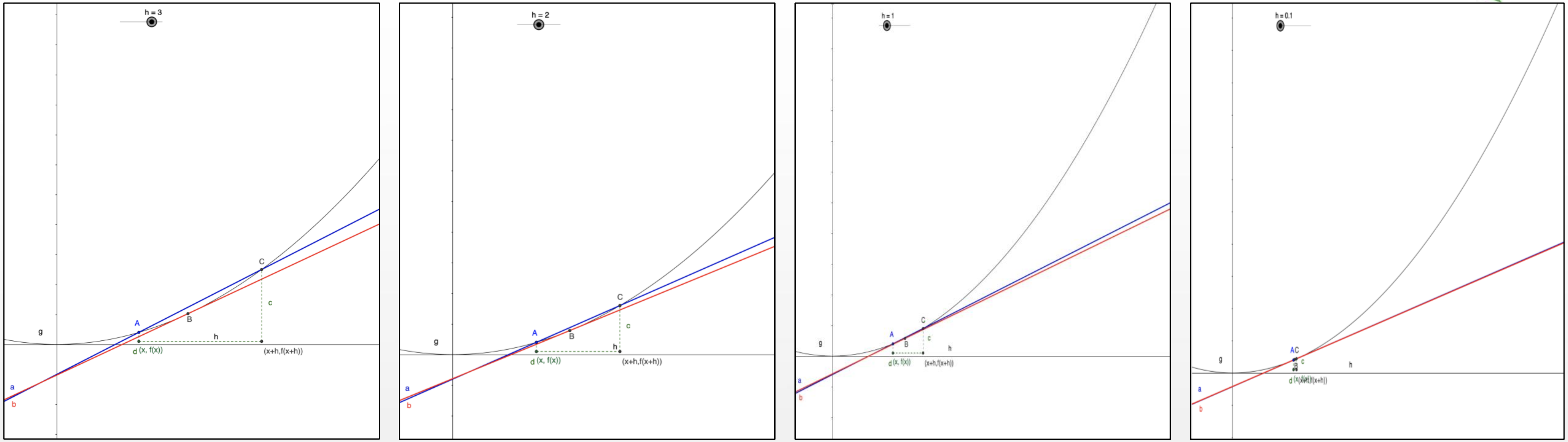
In the diagram on the right can you identify the tangent to the curve, g?

The co-ordinates of *A* are $(x, f(x))$ and the co-ordinates of *C* are $(x+h, f(x+h))$.

Using your knowledge of co-ordinate geometry write down the slope of the blue line, *AC*.

Now let's see what happens as the value of h begins to decrease.

Differentiation



As you look at the images as h decreases from 4 towards 0 can you explain in your own words what is happening to line AC as h gets smaller?

Differentiation from First Principles

From this task we should now see that as *h* approaches zero, the line *AC* is becoming the tangent to the curve at the point *A*. The slope of this line is then given by:

$$\lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

This was the original method used to differentiate a function and although we have simpler methods today, we still sometimes use this approach which is known as differentiation from first principles.

Differentiation by First Principles

Given the linear function $y = 3x + 6$ we can easily find the slope of this line since $y = mx + c$ and so $m = 3$.

However, if we use the method just derived to find the slope we will see it still holds:

$f(x) = 3x + 6$

$f(x + h) = 3(x + h) + 6 = 3x + 3h + 6$

$f(x + h) - f(x) = 3x + 3h + 6 - 3x - 6 = 3h$

$$\frac{f(x+h)-f(x)}{h} = \frac{3h}{h} = 3$$

Hence, the slope = derivative = 3.

An Irish Link

Isaac Newton's Original Method

Suppose $f(x) = x^3$

Then the average rate of change with respect to x over the interval $[x, x + h]$ is

$$\frac{f(x+h) - f(x)}{h} = \frac{(x+h)^3 - x^3}{h}$$

$$\Rightarrow f'(x) = \frac{3hx^2 + 3xh^2 + h^3}{h}$$

At this stage we will **ignore** h^2 and h^3 as they are so small.

So now we can say: $f'(x) = \dfrac{3x^2 h}{h}$

$= 3x^2$

The Irish Contribution

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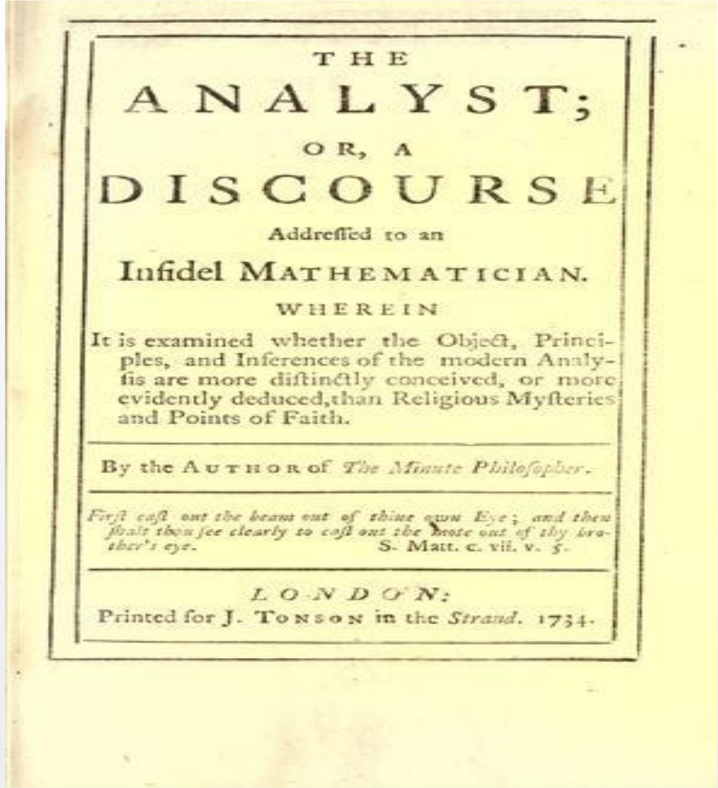

THE
ANALYST;
OR, A
DISCOURSE
Addreffed to an
Infidel MATHEMATICIAN.
WHEREIN
It is examined whether the Object, Principles, and Inferences of the modern Analyfis are more diftinctly conceived, or more evidently deduced, than Religious Myfteries and Points of Faith.

By the AUTHOR of *The Minute Philofopher.*

Firft caft out the beam out of thine own Eye; and then fhalt thou fee clearly to caft out the mote out of thy brother's eye. S. Matt. c. vii. v. 5.

LONDON:
Printed for J. TONSON in the *Strand.* 1734.