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Trigonometry

Teacher CPD Overview

Trigonometry Lessons & Teacher CPD Overview

	Lessons	Teacher CPD Focus
1	Triangles & Right-Angled Triangles Exploration	
2	Pythagoras Theorem (Recap)	Pythagoras' Theorem – Common Misconceptions/ Mistakes (Involving Proof & Calculations)
3	Right Angled Triangle – Relationship between Sides & Angles (Toothpick/Ladder Activity)	Transforming Textbook Questions → Authentic Problems
4	Introduction to Sin/Cos/Tas as a Function	Sin/Cos/Tan → Similar Triangles
5	Sin/Cos/Tan → Ratio of Sides	
6	Solving Trigonometric Equations → To Find Side	
7.	Inverse Trigonometric Functions ➔ To Find Angle	Inverse Functions → Other Types/ Link between Trig Functions & Inverse Trig Functions
8	Learning Experience: Real-Life Problem (Exploration of a Problem – No definite answer)	Link to the Problem-Solving Cycle + Potential CBAs (Using Learning Experiences to build to a CBA)

Teacher CPD & Types of Knowledge

	Common Content Knowledge	Specialised Content Knowledge	Knowledge of Content & Students	Knowledge of Content & Teaching	Knowledge of Curriculum
1.	✓		✓		
2.	✓			✓	✓
3.		✓		✓	
4.		✓	✓	✓	
5.			✓	✓	✓

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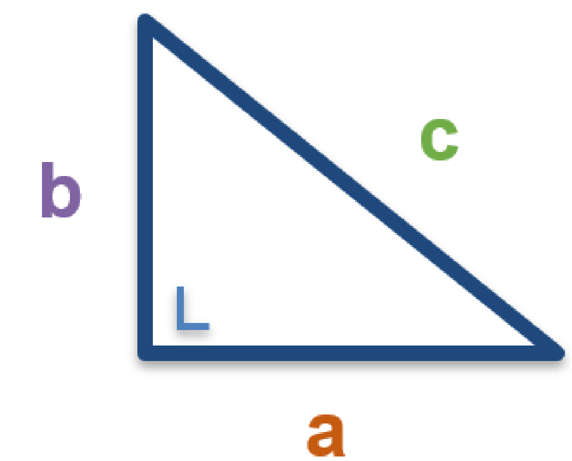
Teacher CPD #1: Pythagoras Theorem
Theorem & Common Misconceptions

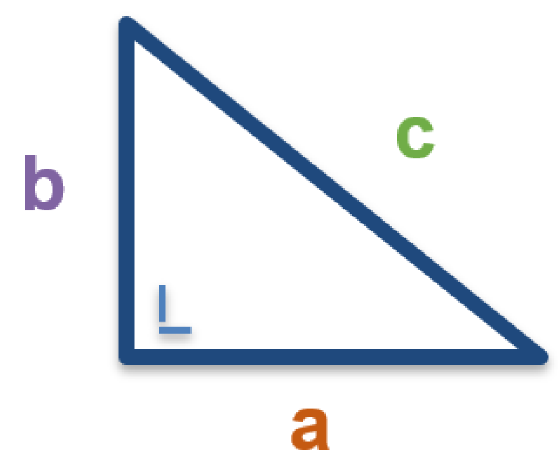
Pythagorean Theorem

- If a triangle is a right angled triangle, then the sum of the squares of the lengths of the legs is equal to the sum of the square of the length of the hypotenuse.

If…	Then…
ΔABC is a right-angled triangle	$(leg_1)^2 + (leg_2)^2 = (hypotenuse)^2$
	$a^2 + b^2 = c^2$

Converse of the Pythagorean Theorem

- If the sum of the squares of the lengths of two sides of a triangle is equal to the square of the length of the third side (the hypotenuse), then the triangle is right angled.

If…	Then…
$(leg_1)^2 + (leg_2)^2 = (hypotenuse)^2$	ΔABC is a right-angled triangle
$a^2 + b^2 = c^2$	

Link to Learning Outcomes

b. recall and use the concepts, axioms, theorems, corollaries and converses, specified in *Geometry for Post-Primary School Mathematics* (section 9 for OL and **section 10 for HL**)

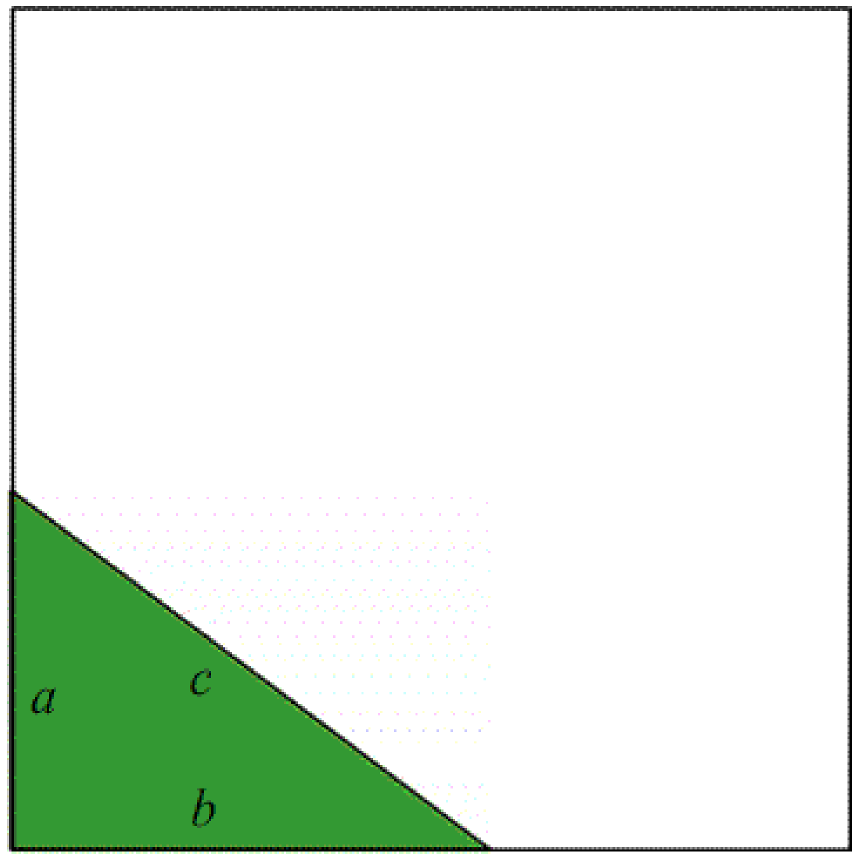
   I. axioms 1, 2, 3, 4 and 5

   II. theorems 1, 2, 3, 4, 5, 6, 9, 10, 13, 14, 15 **and 11, 12, 19,** and appropriate converses including relevant operations involving square roots

   III. corollaries 3, 4 **and 1, 2, 5** and appropriate converses

c. use **and explain** the terms: theorem, proof, axiom, corollary, converse, and implies

Proof – Pythagoras Theorem

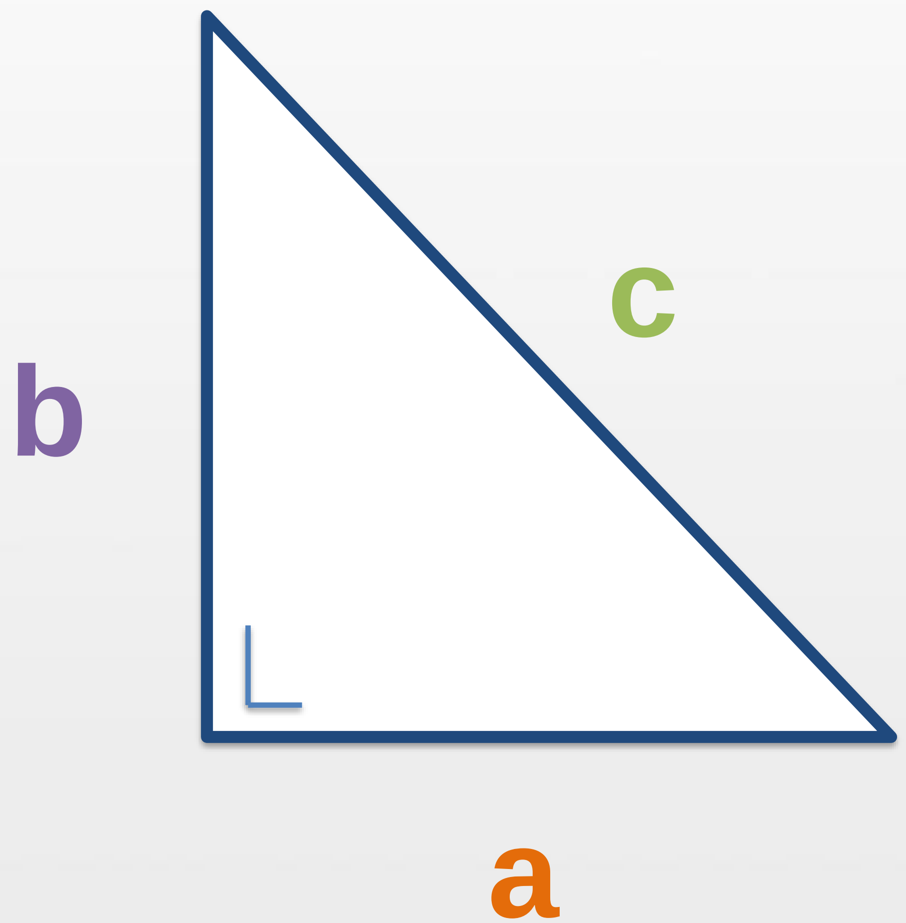


A right triangle, with legs a and b and hypotenuse c.

Pythagorean Triple

- A Pythagorean triple consists of three positive integers a, b and c, such that $a^2 + b^2 = c^2$
- When a triangle's sides are a Pythagorean Triple, it is a right angle triangle.

Pythagorean Triple	
(3, 4, 5)	(5, 12, 13)
(7, 24, 25)	(8, 15, 17)
(9, 40, 41)	(11, 60, 61)
(15, 20, 25)	Infinite more…

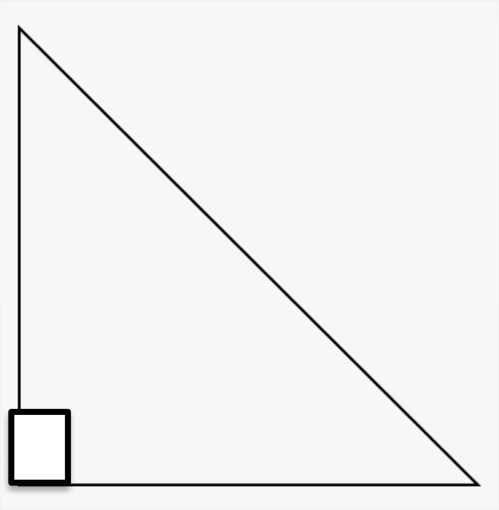


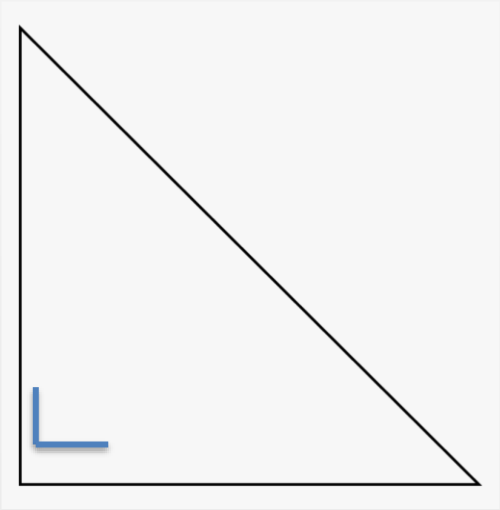
Common Student Misconceptions/ Mistakes

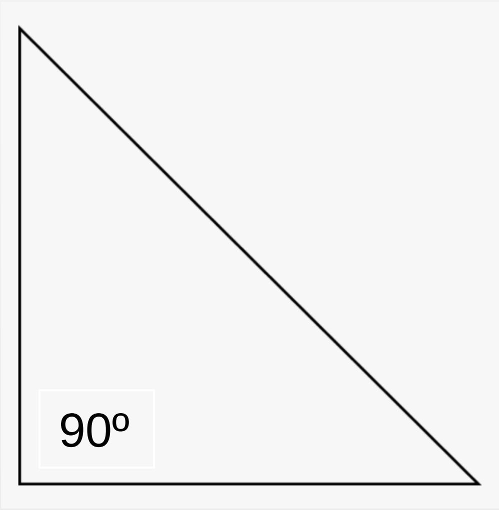
- Recognising right-angled triangles
- Labelling the sides
- Dividing by two
- Finding a side other than the hypotenuse
- Not seeing the connection between the theorem and its converse.

Students may not recognise a right-angle triangle

- Students sometimes struggle to identify if a triangle is right angled or not.
- This results in students using Pythagoras theorem incorrectly on non-right angled triangles.
- Important that students are aware of the different ways that right angled triangles can be labelled.



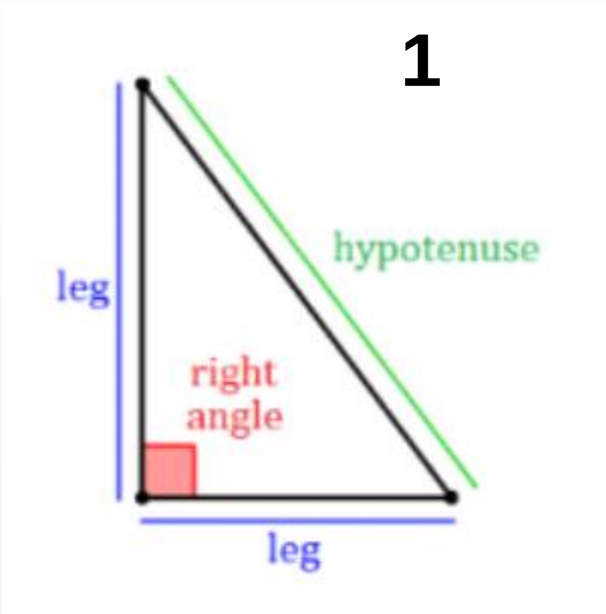




Labelling the triangle – Which side is which?

- Students often struggle with the labelling of a right angled triangle.
- Often what is perceived to be the longest side is labelled the hypotenuse. However, this might not actually be the longest side.
- It is important that students know the hypotenuse is across from the right angle.
- This can cause confusion when triangles are presented differently.

1



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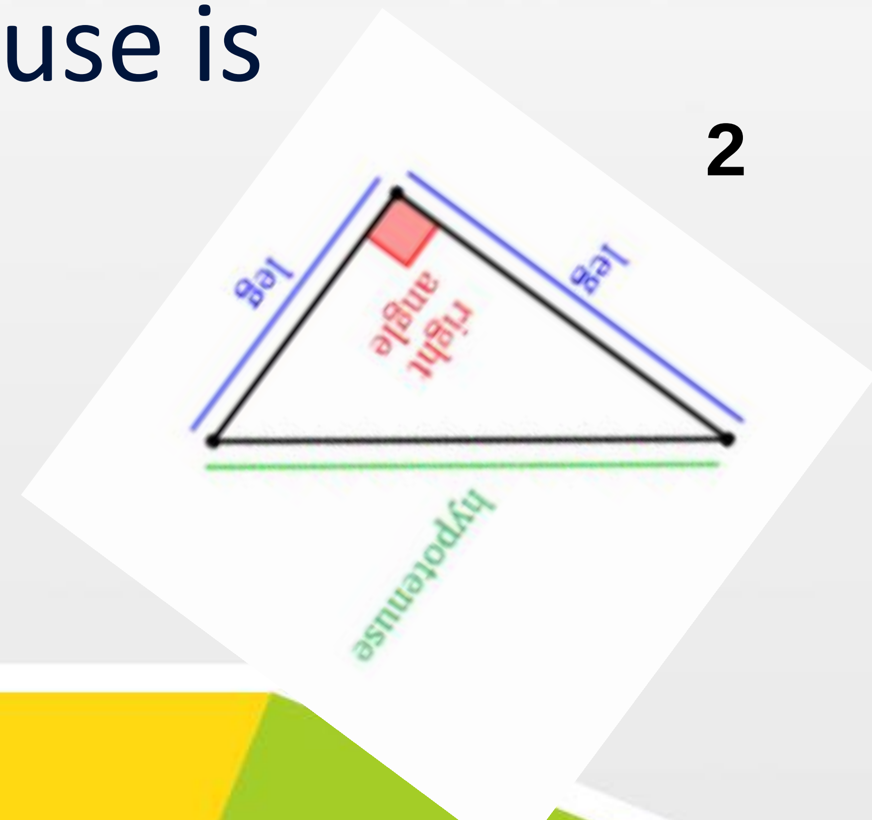
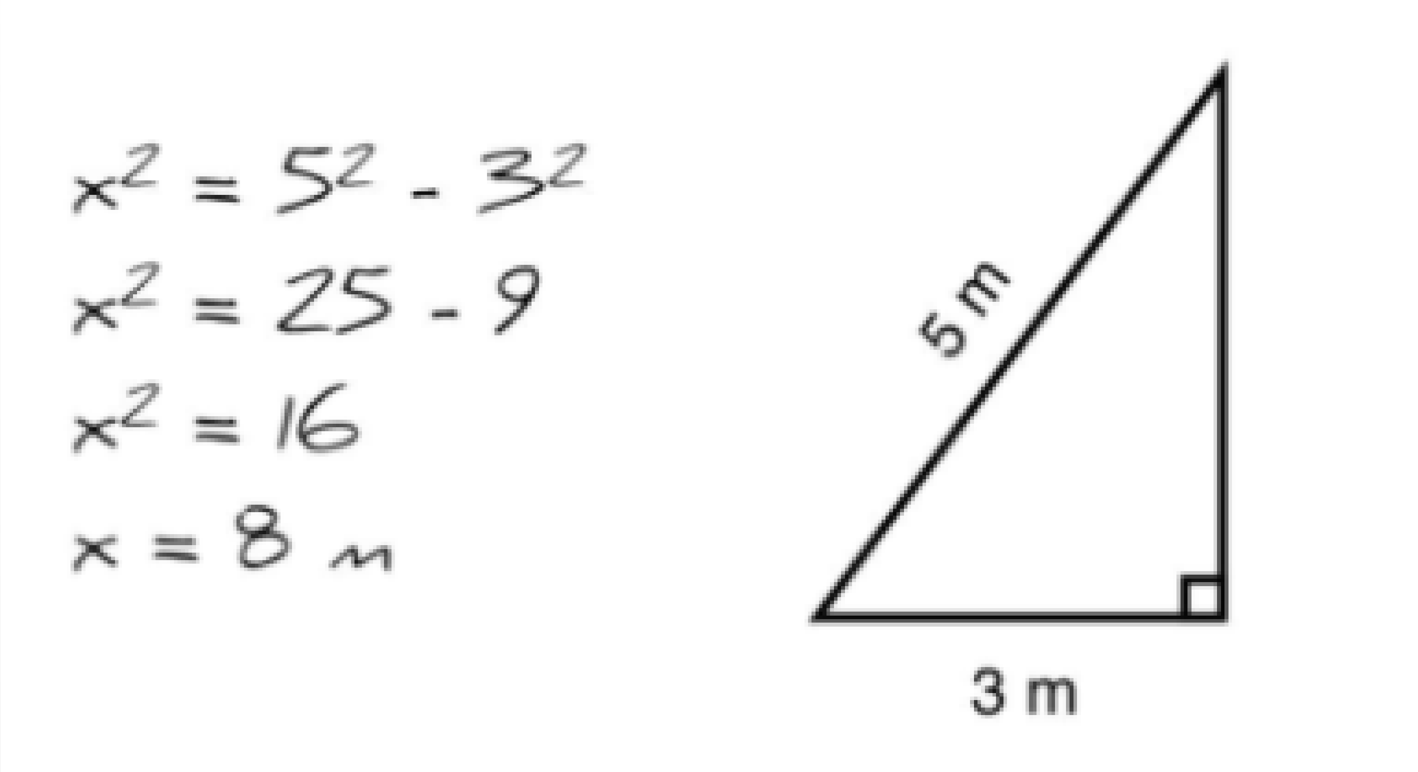


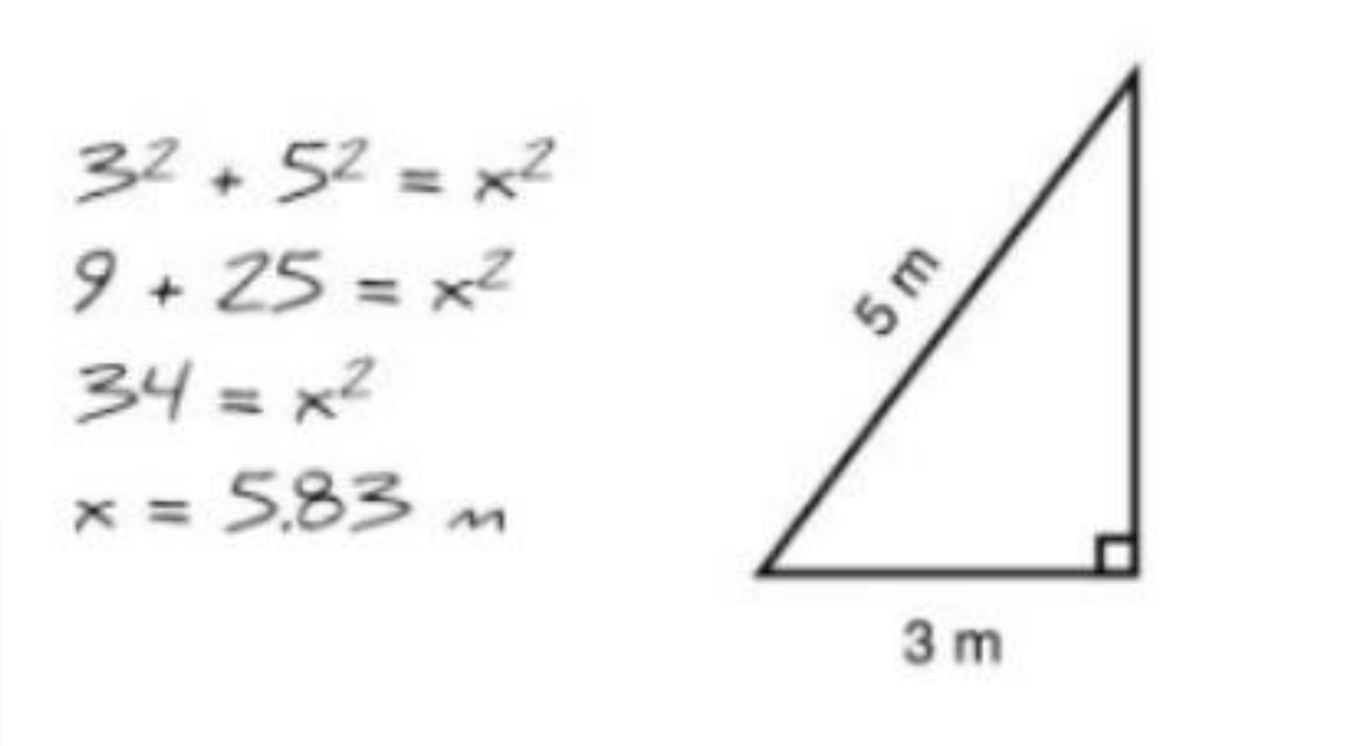
Dividing by 2 instead of finding the square root

- $(hypotenuse)^2 = (leg_1)^2 + (leg_2)^2$
- Students often have a misunderstanding surrounding the power of two and square root operations.
- Students sometimes multiply by 2 instead of squaring.
- Similarly, students divide by 2 instead of finding the square root.
- Explaining these operations as inverse operations of each other should help increase understanding.



Finding a side other than the hypotenuse

- Students sometimes assume that the unknown side is always the side that is isolated in the Pythagoras theorem formula.
- $(hypotenuse)^2 = (leg_1)^2 + (leg_2)^2$
- As a result, students sometimes automatically put in the unknown value for the hypotenuse, when in fact it is one of the legs of the triangle.
- It needs to be reinforced that the hypotenuse is the longest side of the triangle and students need to be familiar with using the formula to find a hypotenuse and other sides of the triangles.



Converse of the Theorem

- The converse of the theorem is equally as important in proving triangles are right-angled.
- Often, students are able to use the theorem but struggle to see the connection between the theorem and its converse.
- Encourage students to look at questions involving…
  - If $(hypotenuse)^2 = (leg_1)^2 + (leg_2)^2$ then the triangle is right angled.
  - If the triangle is right angled, then $(hypotenuse)^2 = (leg_1)^2 + (leg_2)^2$
- This will help increase understanding of not only Pythagoras theorem in trigonometry but a wider knowledge of theorems and their converses in geometry.

Reflection:

- How did you approach teaching Pythagoras before?
- How might your practice change as a result of this video?
- Is there anything from this video that you would like to learn more about?

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References:

- https://www.jct.ie/maths/planning_resources
- https://commons. File:Pythagorean_Theorem_Proof.gif
- https://www.onlinemathlearning.com/pythagorean-triples.html

UNIVERSITY OF LIMERICK
OLLSCOIL LUIMNIGH
School of Education

Contact details

Web Link: https://epistem.ie

Twitter handle

Email:

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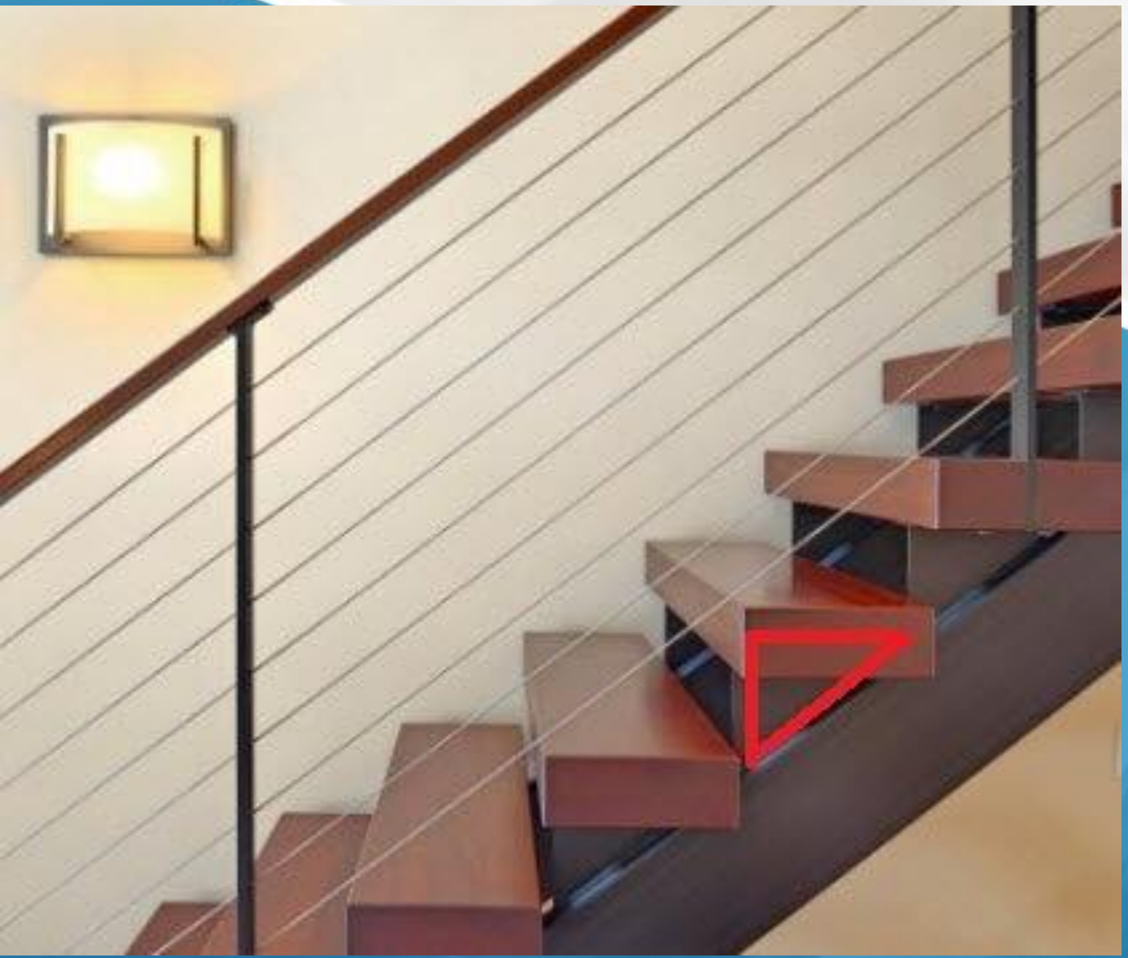
Trigonometry

Teacher CPD #2: Transforming Textbook Questions

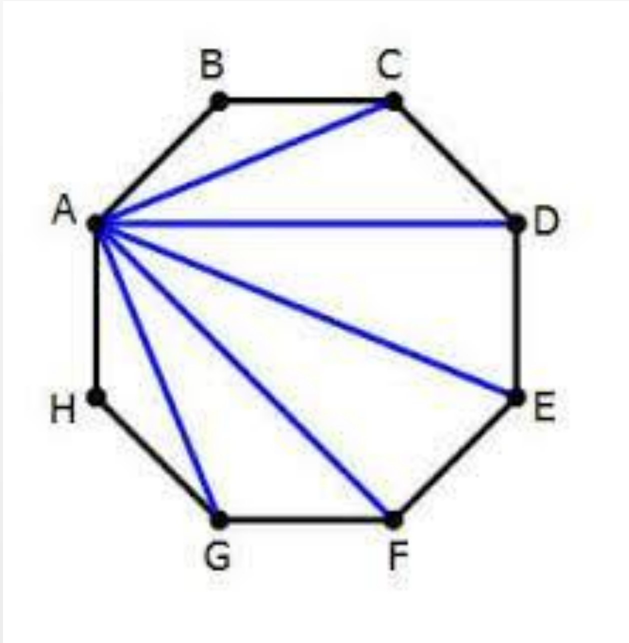
Creating Authentic Questions





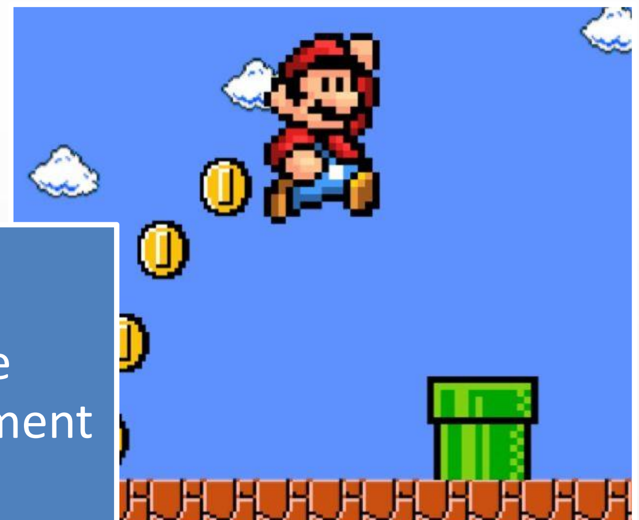

Finding Right Angled Triangles

- Students often have difficulty seeing triangles within various shapes and real-life objects.
- Actively asking the students to identify and create triangles and right-angled triangles is a useful skill that students could practice.

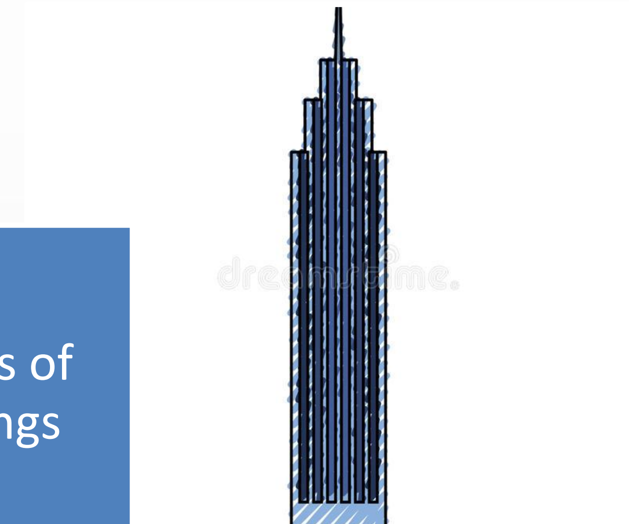


Real-Life Applications

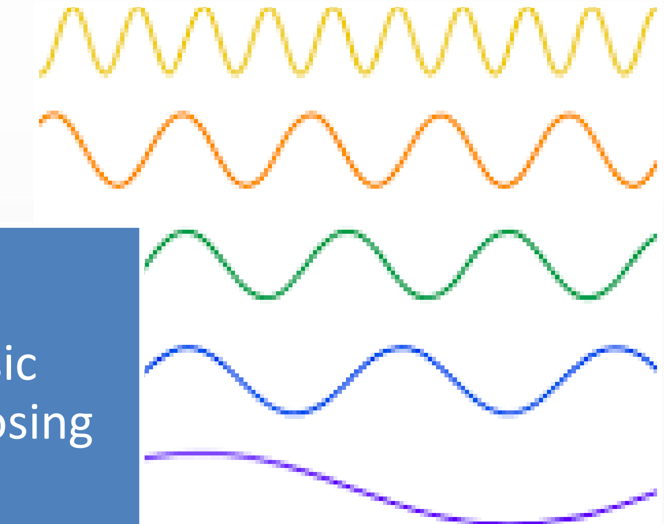
Game Development



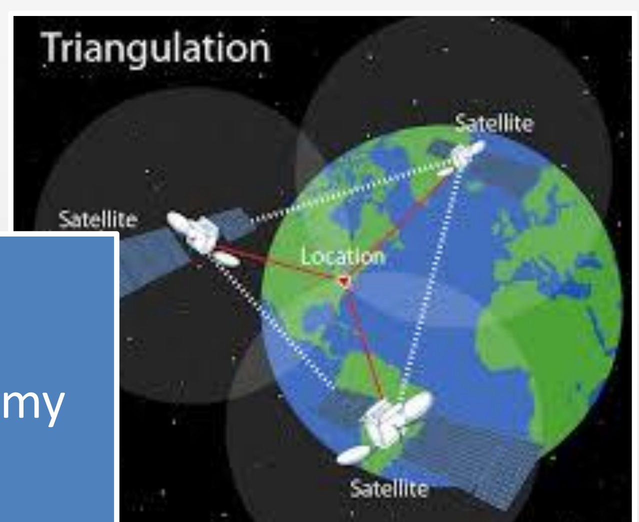
Heights of Buildings



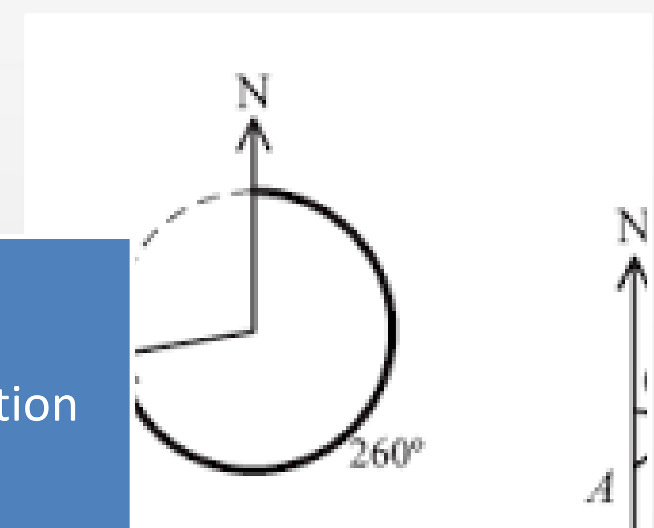
Music Composing



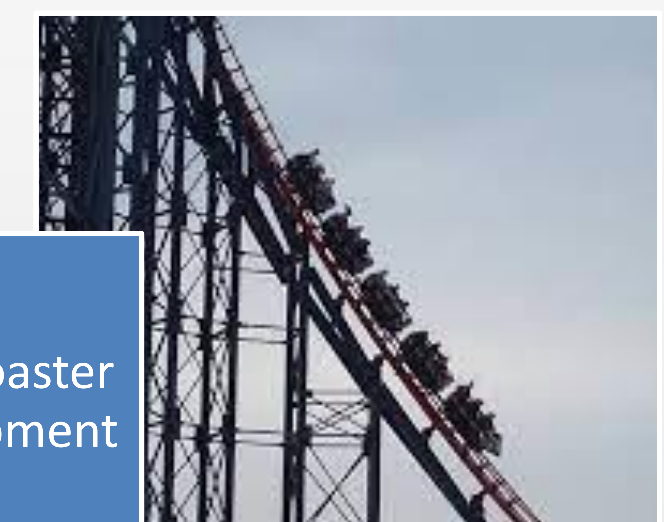
Astronomy



Navigation



Rollercoaster Development



Creating Authentic Questions

- It's important for students to be able to see the real-life application of these questions from the beginning, and not necessarily something that is introduced at the end.
- This can be done by using real-life examples, instead of just repeating right-angled triangles.

Creating Authentic Questions

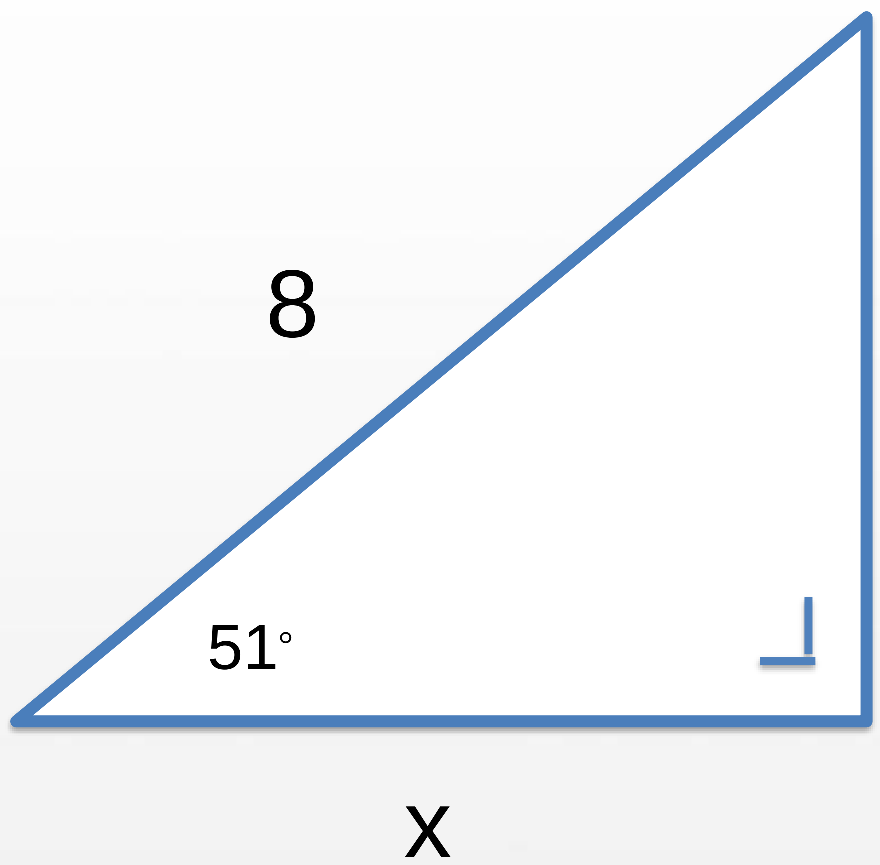
- To create authentic questions, textbook questions can be used and adapted into real-life contexts.
- Terms such as angle of elevation and angle of depression should be introduced from the start and used throughout all real-life examples.
- Also, using terms such as vertical/ horizontal give the students a real-life understanding of right angled triangles.

Angle of Depression & Elevation

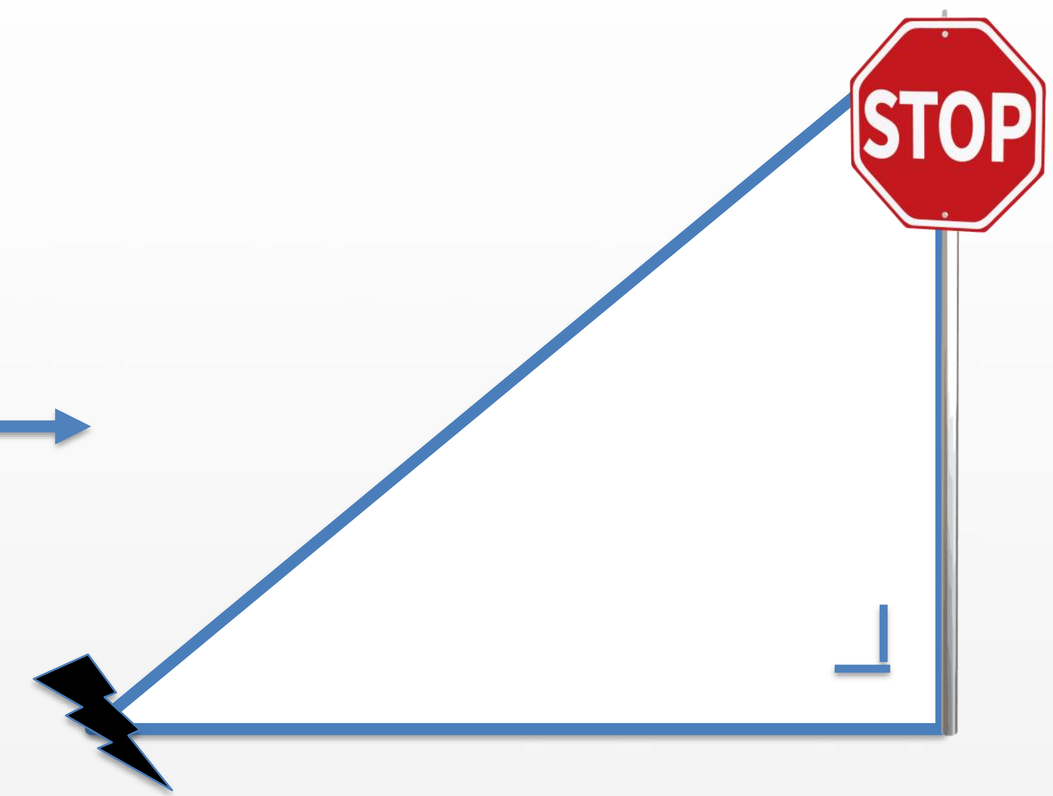
A practical application of the trigonometric functions is to find the measure of lengths that you cannot measure. Very frequently, angles of depression and elevation are used in these types of problems.

Angle of Depression: The angle measured from the horizon or horizontal line, down.

Angle of Elevation: The angle measure from the horizon, or horizontal line, up.



Find x

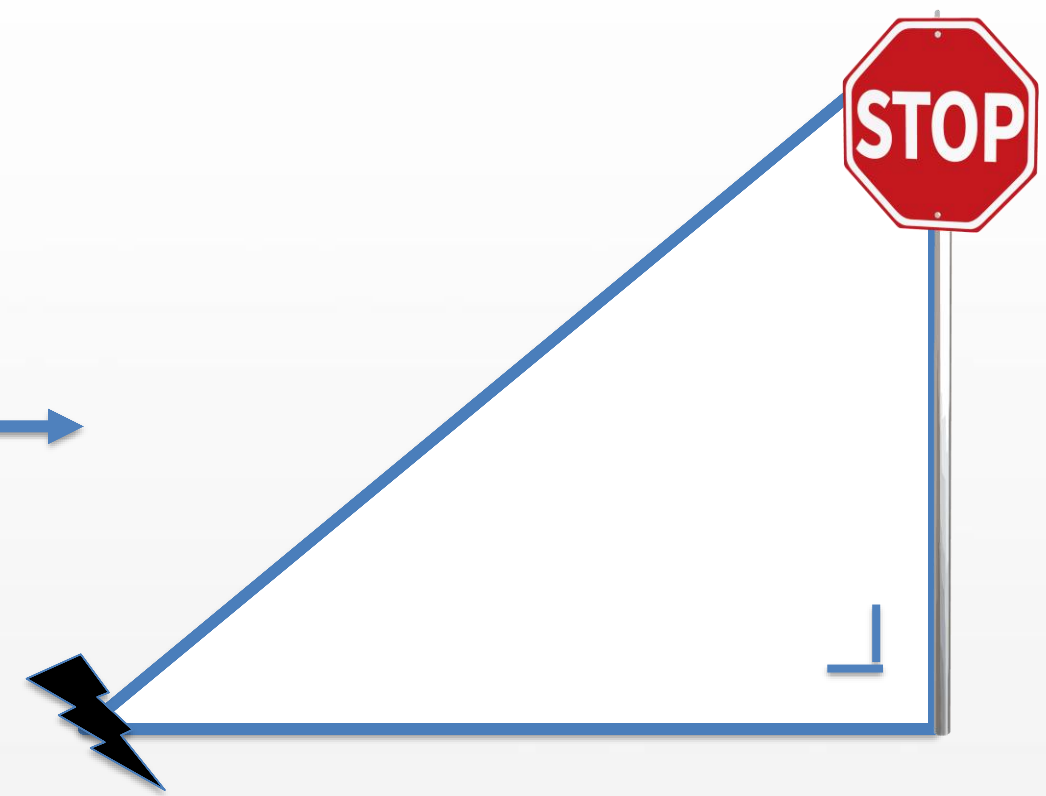


An 8m metal wire is attached to a broken stop sign to secure it in a vertical position until repairs can be made. Attached to a stake in the ground, the wire makes an angle of 51° with the ground. How far from the foot of the stop sign is the stake?

What type of people would be solving problems like this?

What other information might have been helpful?

What tools do you think they might have needed to collect enough information to solve this problem?



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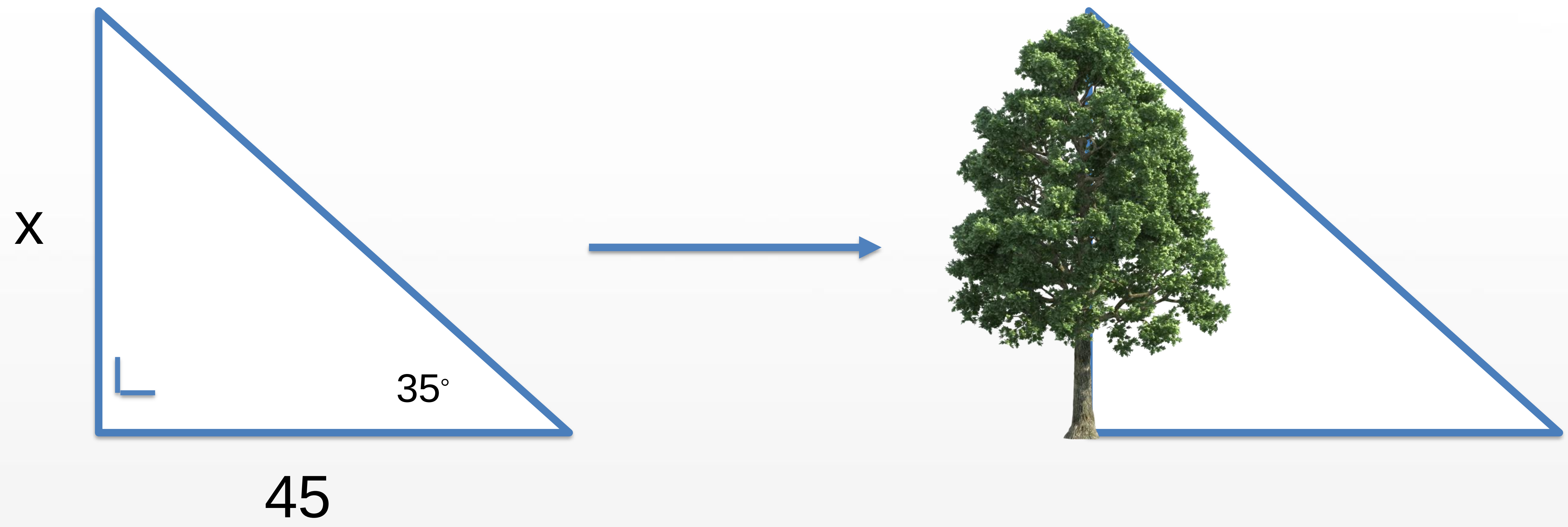


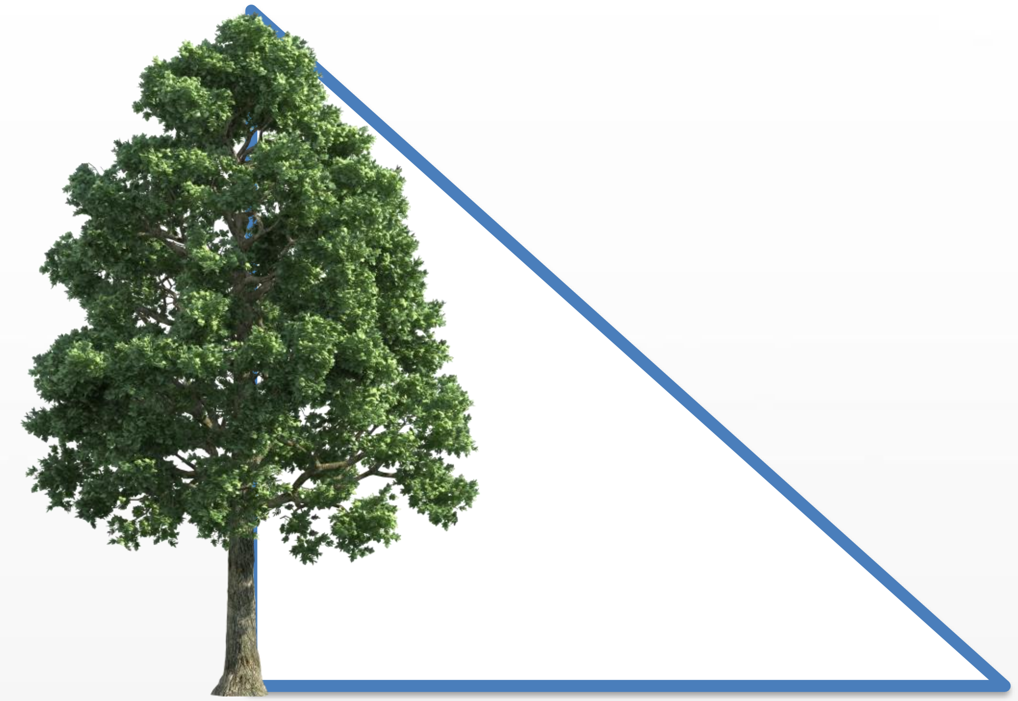
Find x

From a point on the ground 45 metres from the foot of a tree, the **angle of elevation** of the top of the tree is 35º. Find the height of the tree to the nearest metre.

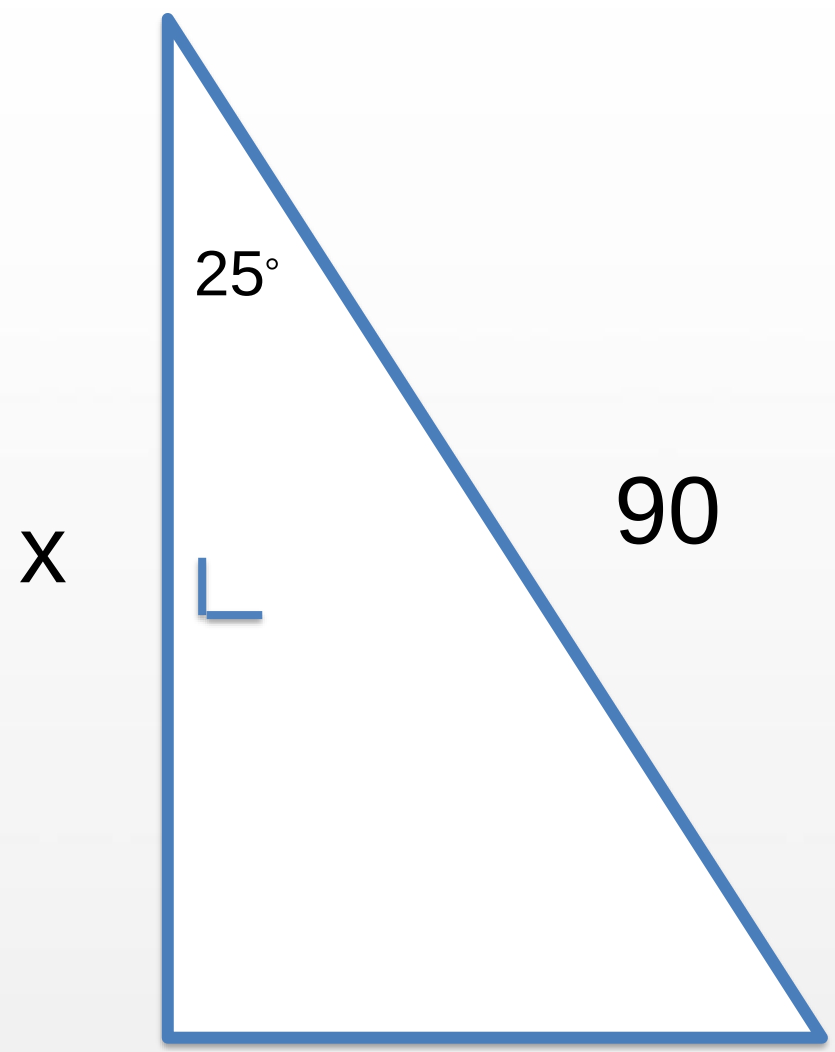
What type of people would be solving problems like this?

What other information might have been helpful?

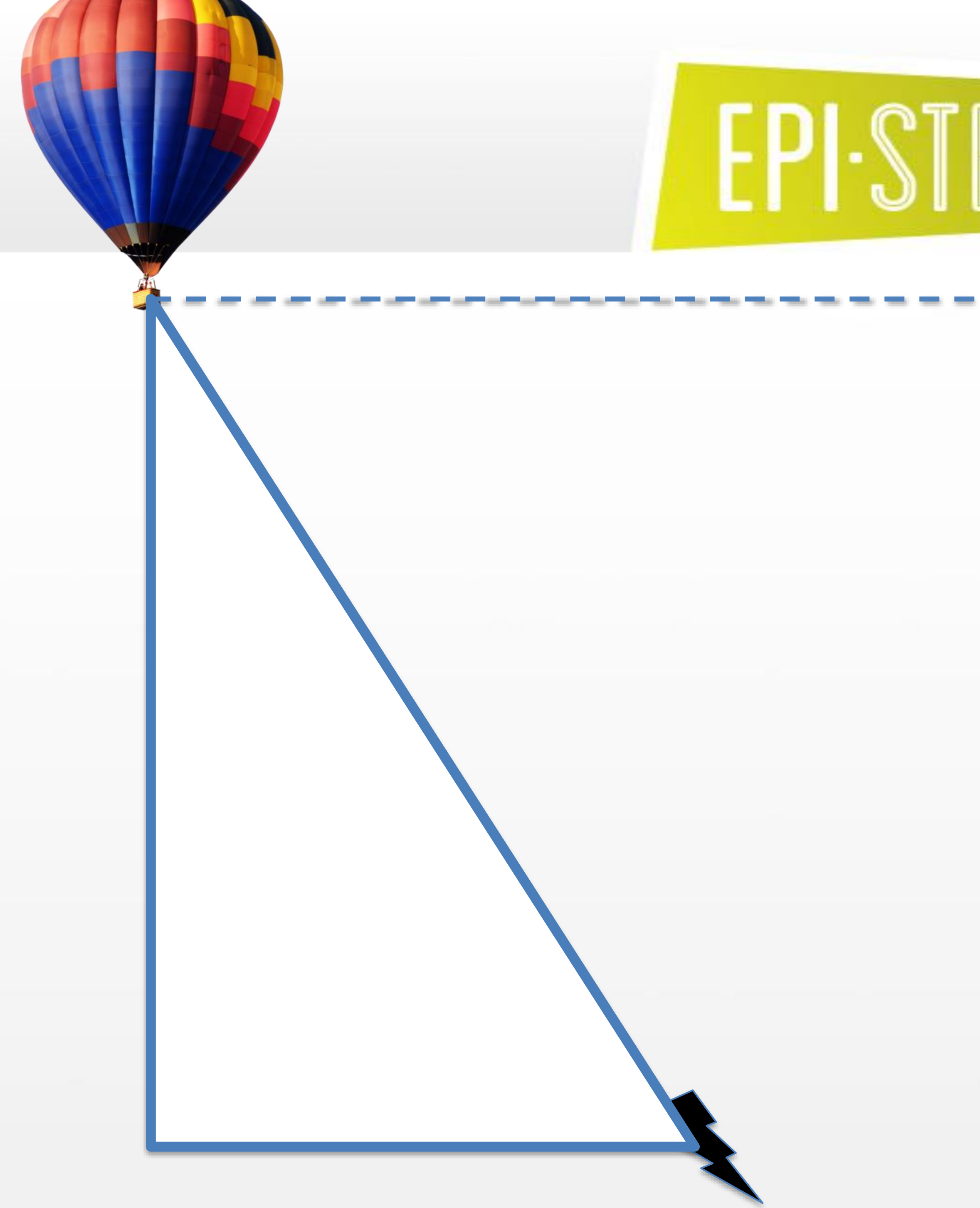
What are the important trigonometric terms that we need to understand to solve this problem?



From a point on the ground 45 metres from the foot of a tree, the **angle of elevation** of the top of the tree is 35º. Find the height of the tree to the nearest metre.



Find x

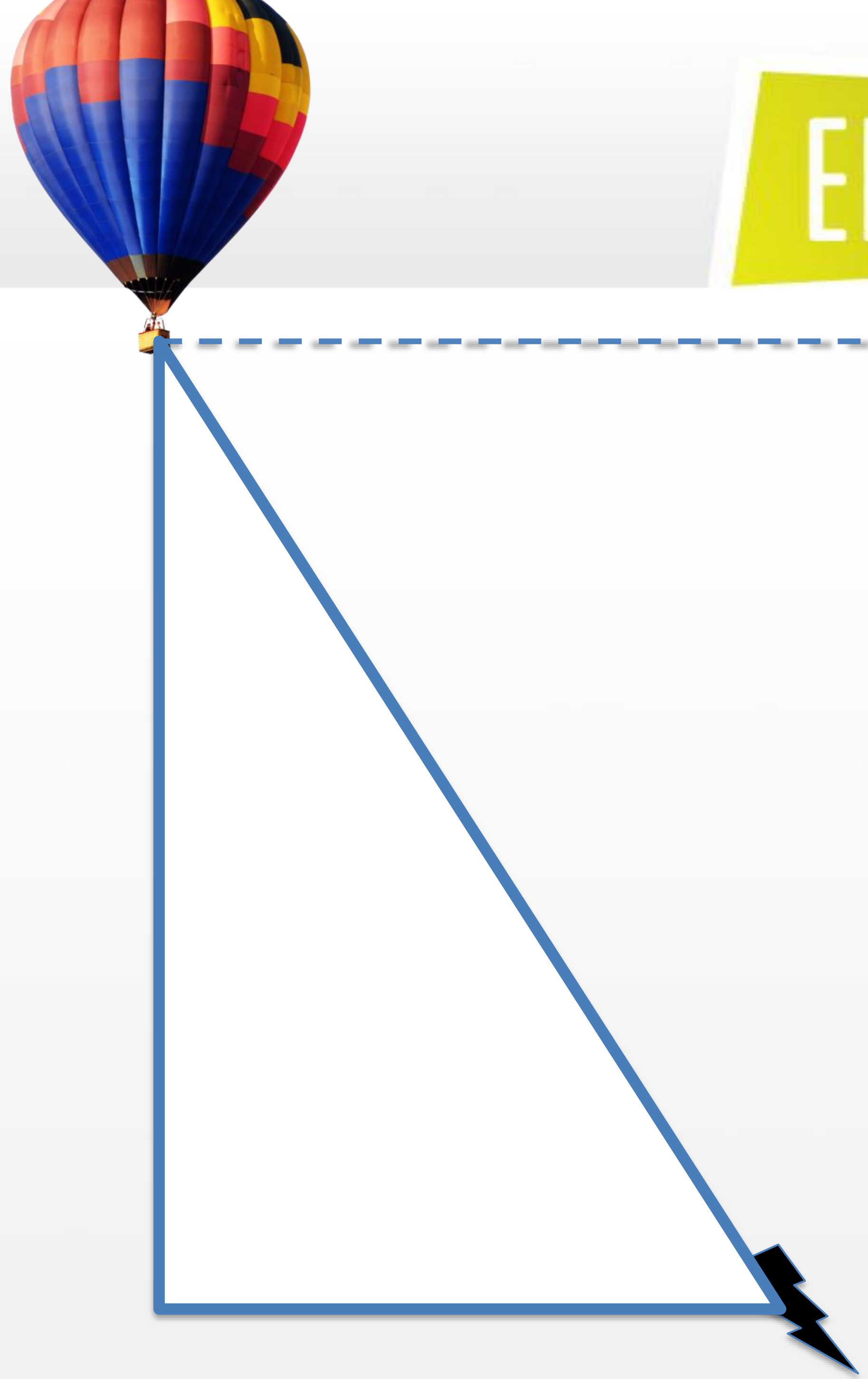


On a windy day, a 90m rope tightly secures a hot air balloon to a stake in the ground. From the balloon, the angle of depression of the stake is 65º. Find the height of the balloon above the ground.

What type of people would be solving problems like this?

What other information might have been helpful?

Why might the angle of depression part of this question confuse some people?



On a windy day, a 90m rope tightly secures a hot air balloon to a stake in the ground. From the balloon, the angle of depression of the stake is 65º. Find the height of the balloon above the ground.

Building Authentic Problems

Using authentic problems in your class will help students:

- Develop problem solving and build fluency skills.
- Help identify and recognise the real-life applications of trigonometry in the world around them.
- Allows the students time to think, reason and build connections between topics and questions.
- Helps students spot patterns, make conjectures and create generalisations.
- Helps students become familiar with the mathematics registrar and the type of communication that is used when dealing with these problems.

Reflection:

- How did you approach teaching real-life applications before? Did you embed them into your lessons from the start?
- How might your practice change as a result of this video?
- Is there anything from this video that you would like to learn more about?

Contact details

Web Link: https://epistem.ie

Twitter handle

Email:

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Trigonometry

Teacher CPD #3: Sin/Cos/Tan Functions

Trigonometric Ratios

- As mathematics teachers, it is important for us to have a content knowledge above what is required for the students.
- A greater conceptual understanding of the content means that a teacher can create greater links and help develop students own conceptual understanding.
- Knowing where trigonometric ratios come from can help explain how these functions are derived. They also provide a link to the Unit Circle which is a central part to Higher Level Leaving Certificate Trigonometry.

Sin/Cos/Tan of an Angle are Line Lengths

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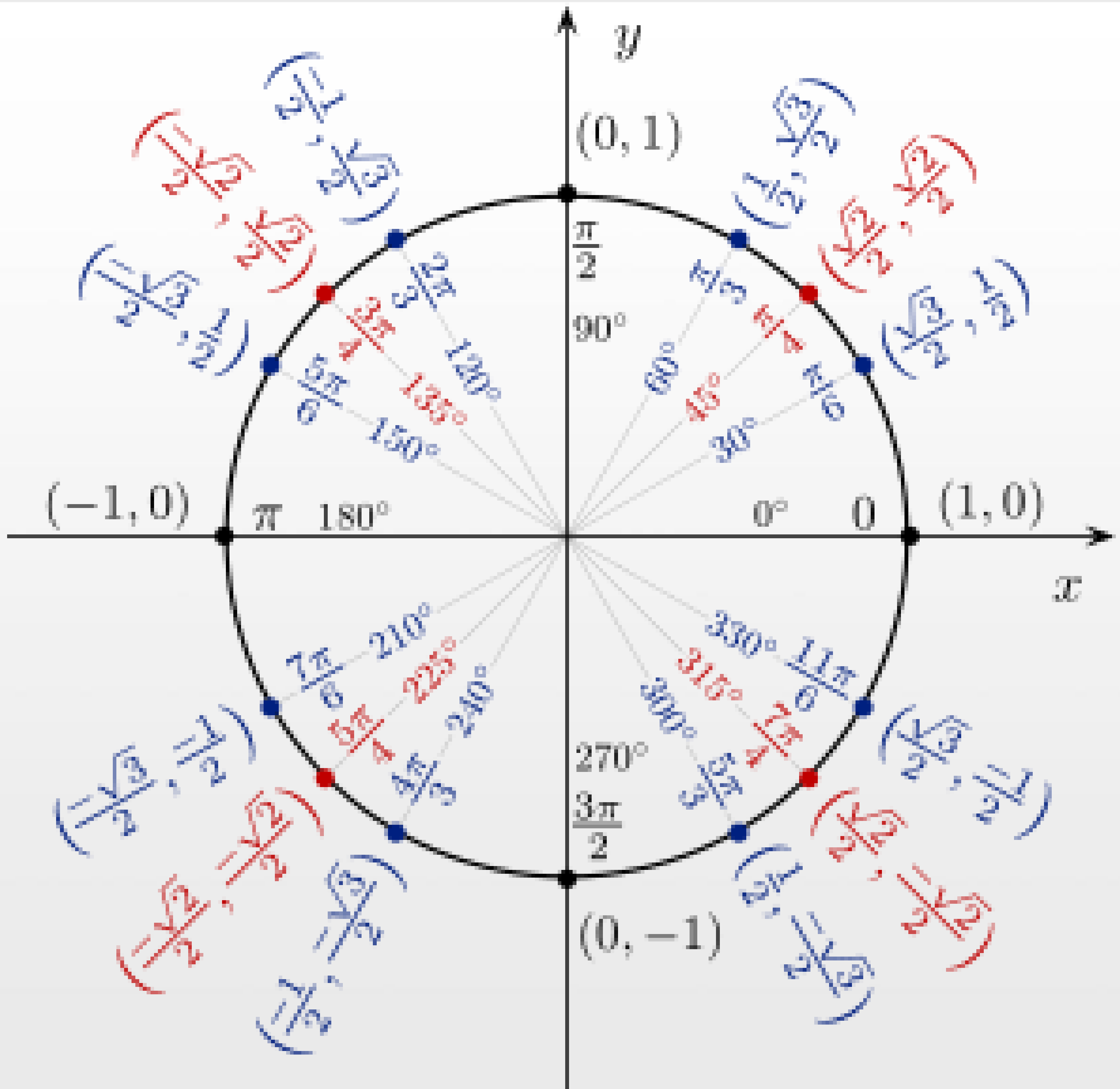
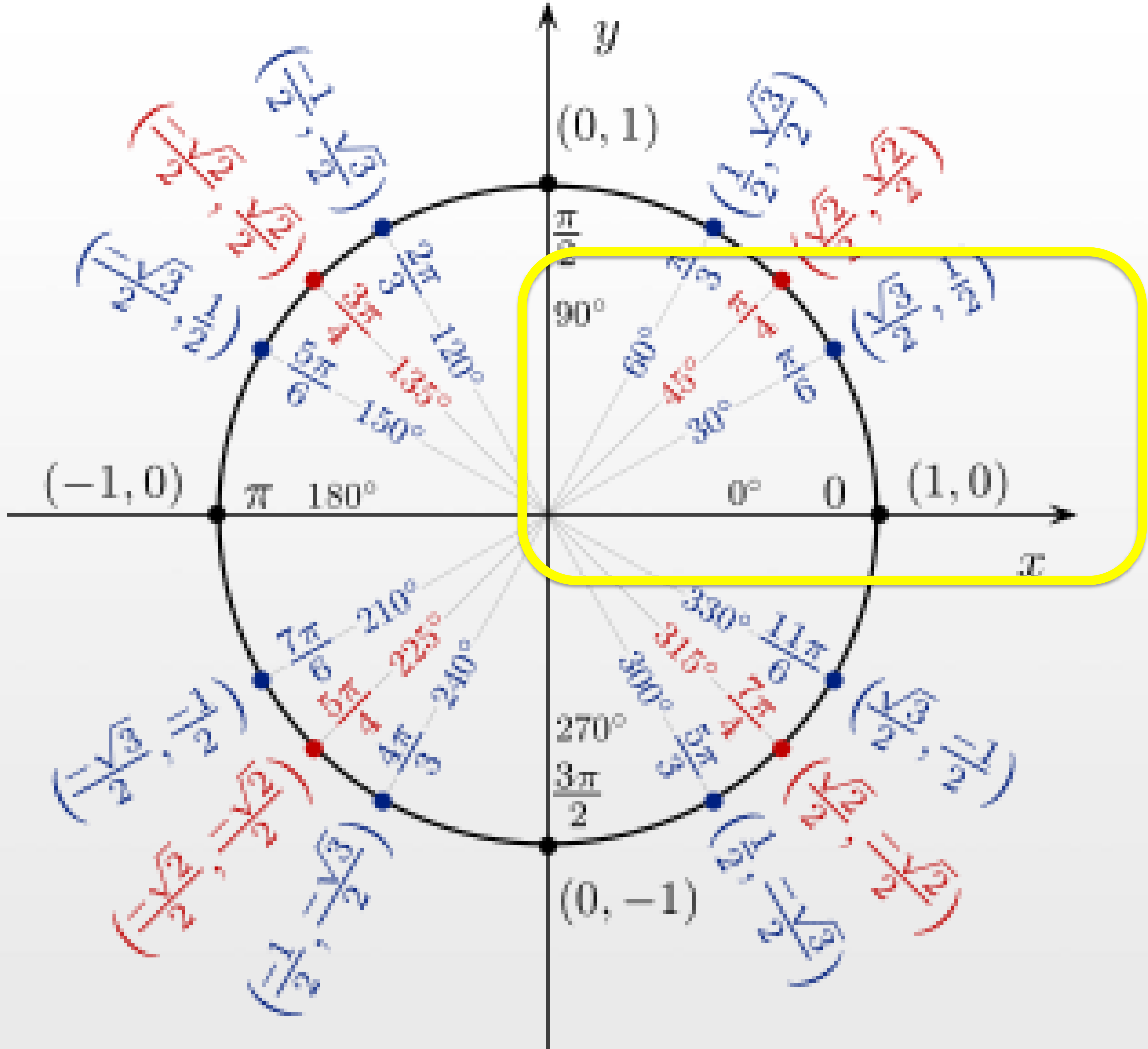
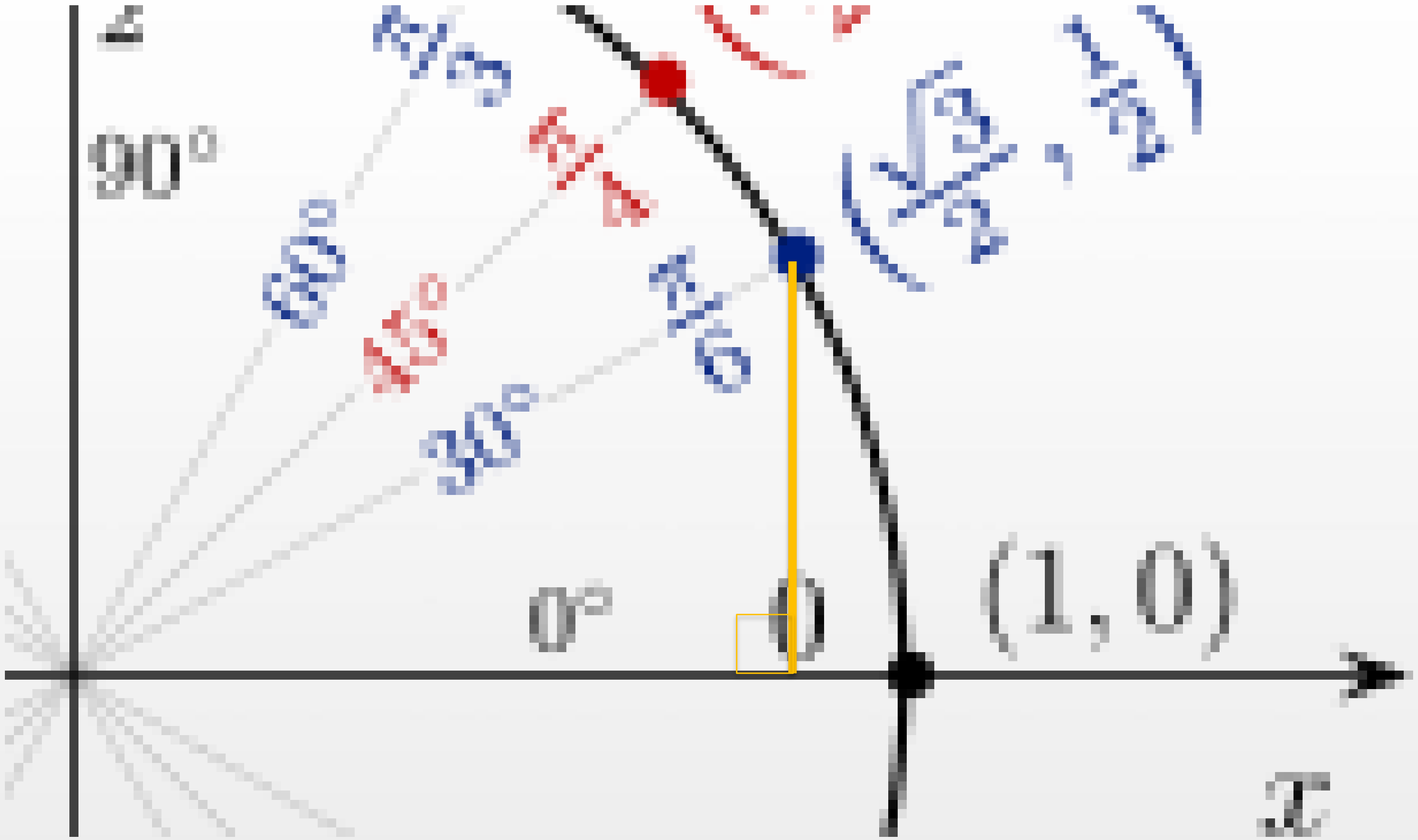


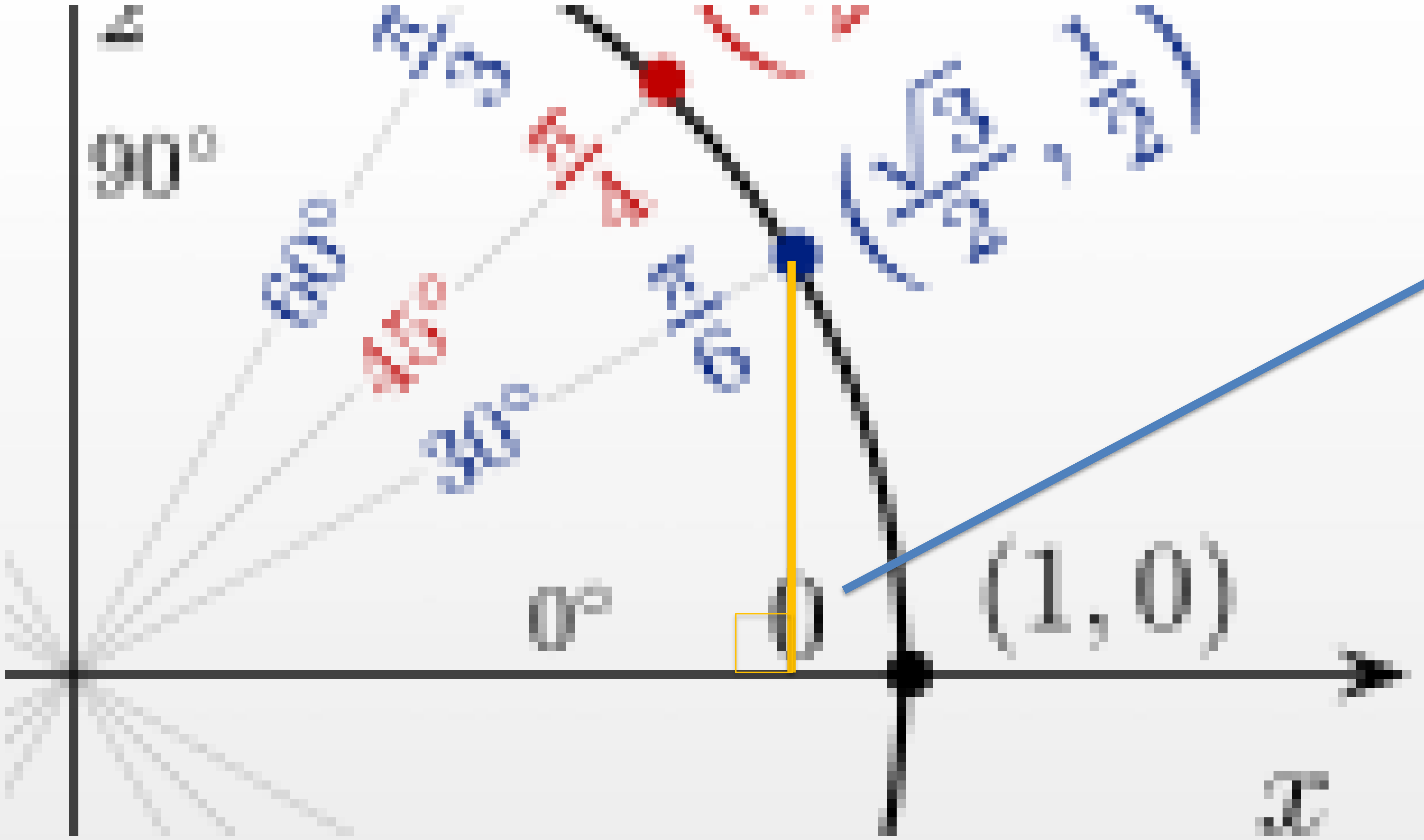
Sin/Cos/Tan of an Angle are Line Lengths



Sin/Cos/Tan of an Angle are Line Lengths



Sin/Cos/Tan of an Angle are Line Lengths



$$Cos30° = \frac{\frac{\sqrt{3}}{2}}{1}$$

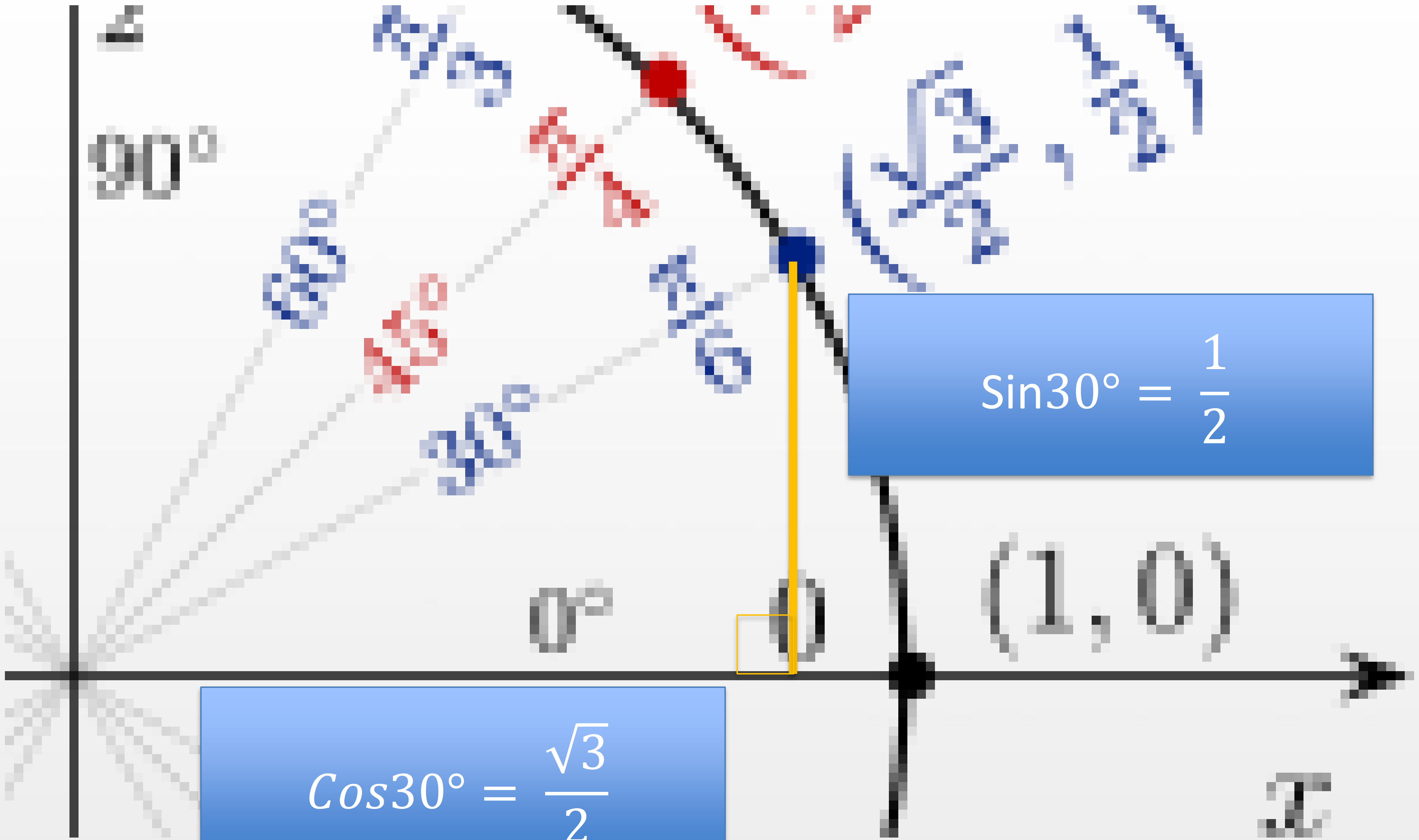
$$Cos30° = \frac{\sqrt{3}}{2}$$

Also,

$$Sin30° = \frac{\frac{1}{2}}{1}$$

$$Sin30° = \frac{1}{2}$$

Sin/Cos/Tan of an Angle are Line Lengths



$\text{Sin}30° = \frac{1}{2}$

$Cos30° = \frac{\sqrt{3}}{2}$

$$Cos30° = \frac{\frac{\sqrt{3}}{2}}{1}$$

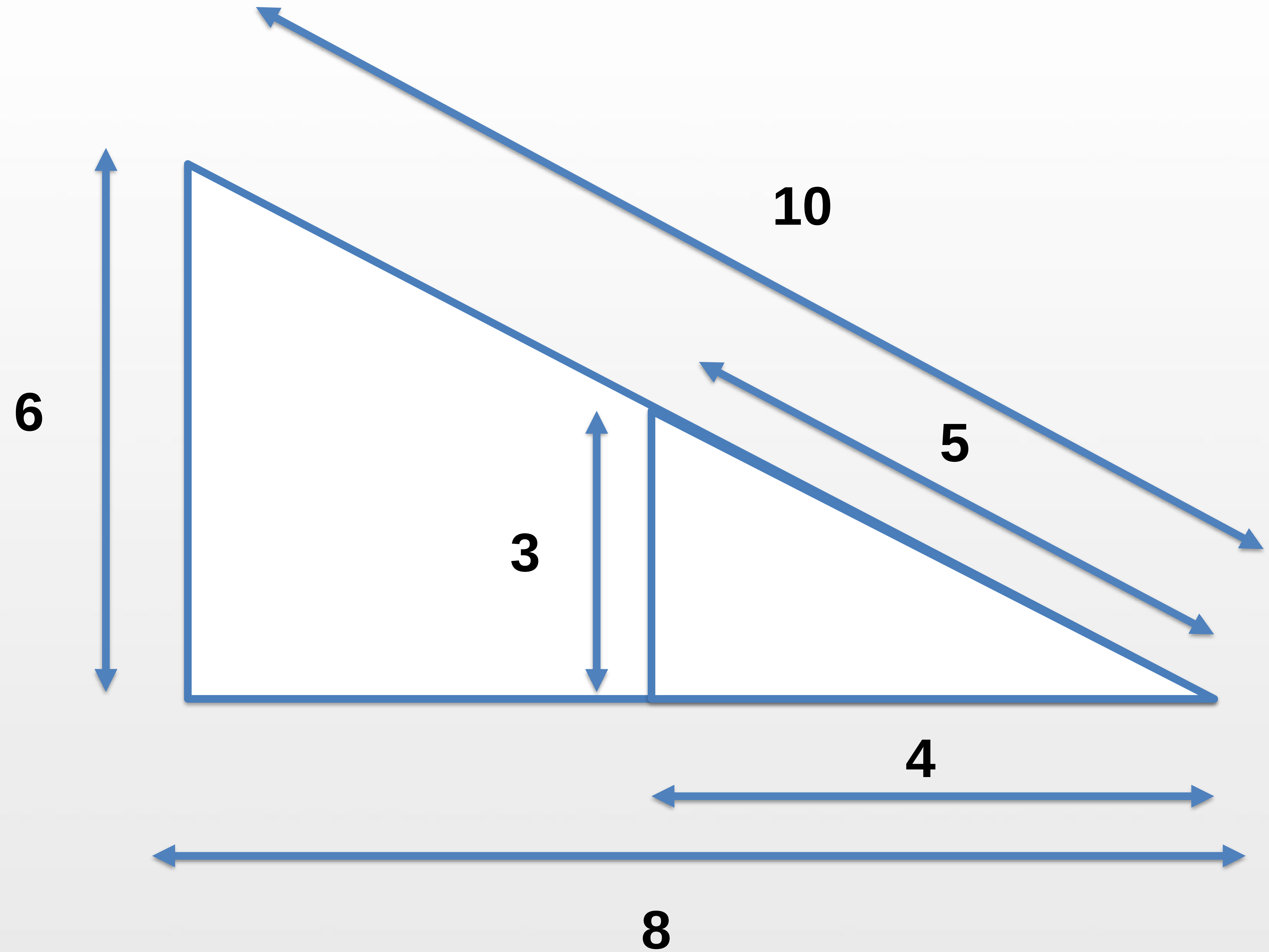
$$Cos30° = \frac{\sqrt{3}}{2}$$

Also,

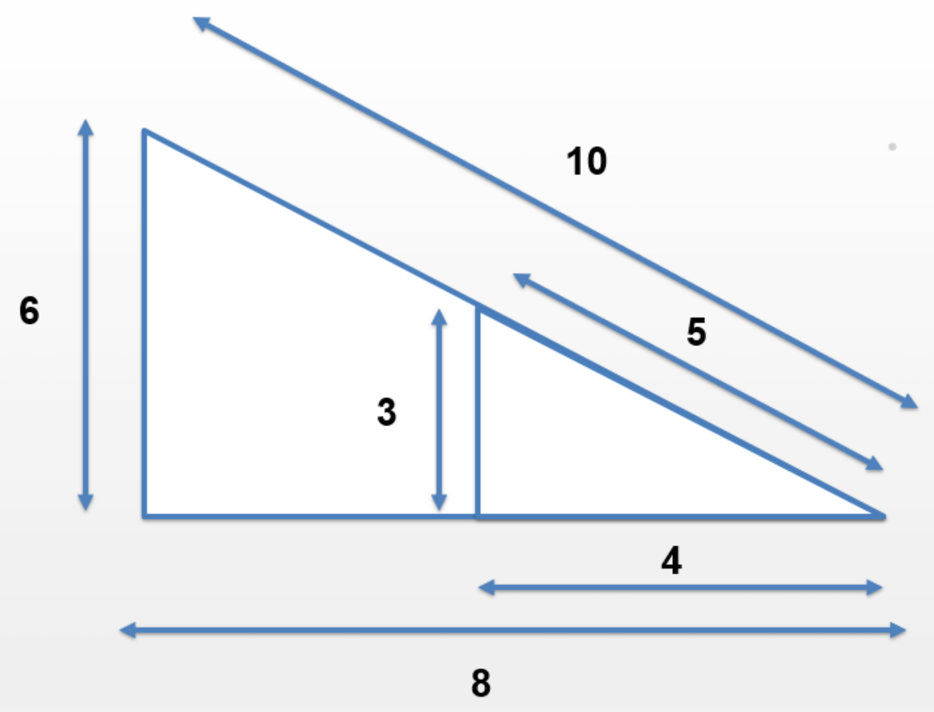
$$Sin30° = \frac{\frac{1}{2}}{1}$$

$$Sin30° = \frac{1}{2}$$

Consider the following right angled triangle..



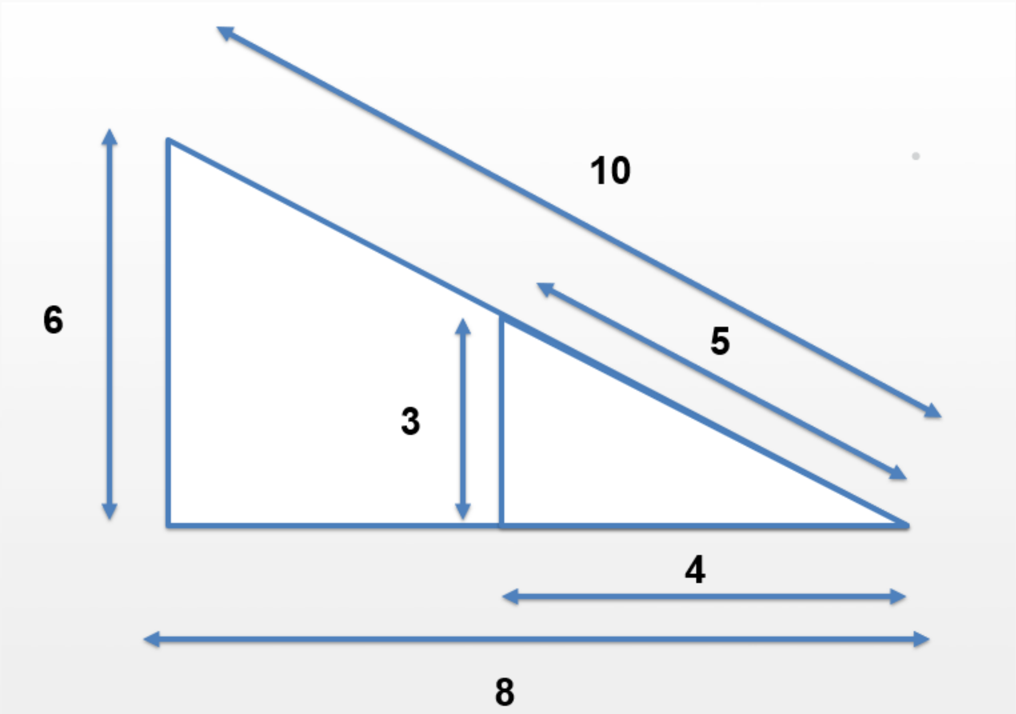
Similar Triangles



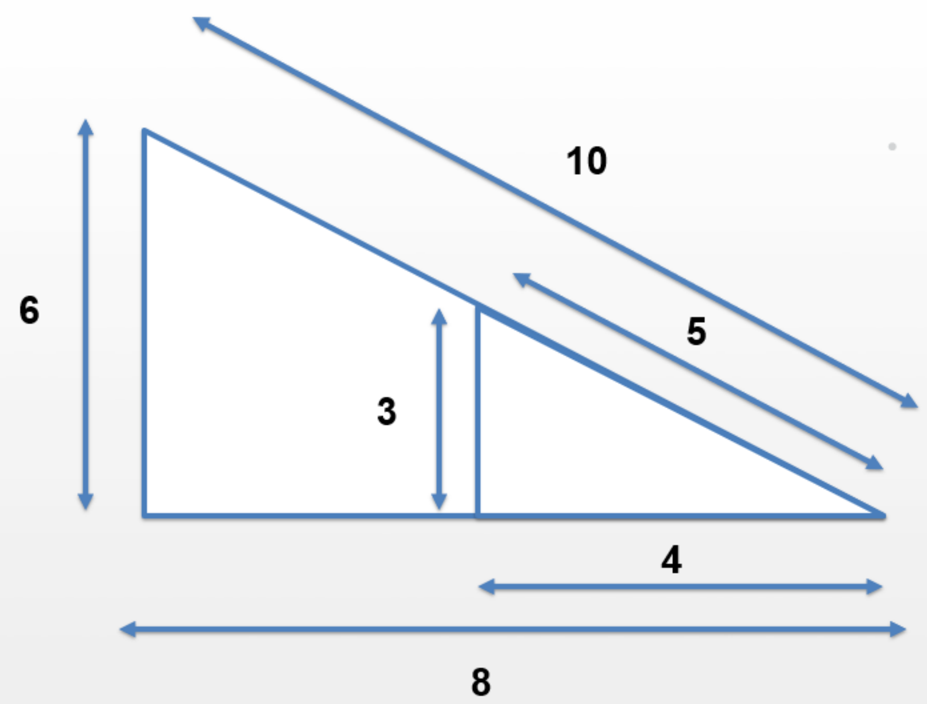
Similar Triangles

Ratio of Corresponding Sides

$$\frac{3}{6} = \frac{5}{10} = \frac{4}{8} \ldots = \frac{1}{2}$$



Similar Triangles



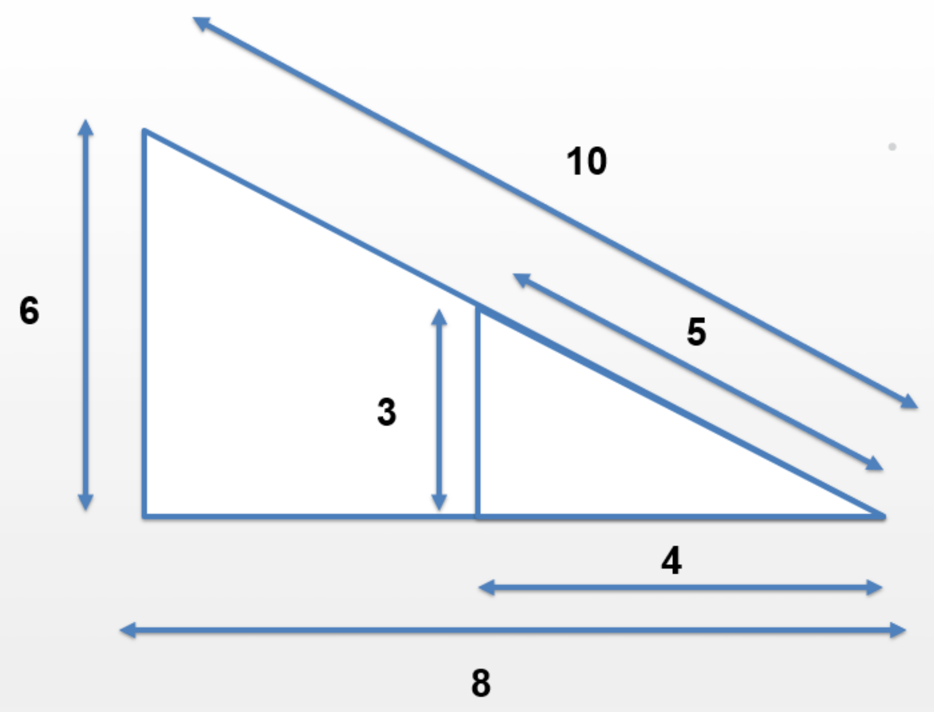
Ratio of Corresponding Sides

$$\frac{3}{6} = \frac{5}{10} = \frac{4}{8} \dots = \frac{1}{2}$$

Ratio of Base to Hypotenuse

$$\frac{8}{10} = \frac{4}{5} \dots = 0.80$$

Similar Triangles



Ratio of Corresponding Sides

$$\frac{3}{6} = \frac{5}{10} = \frac{4}{8} \ldots = \frac{1}{2}$$

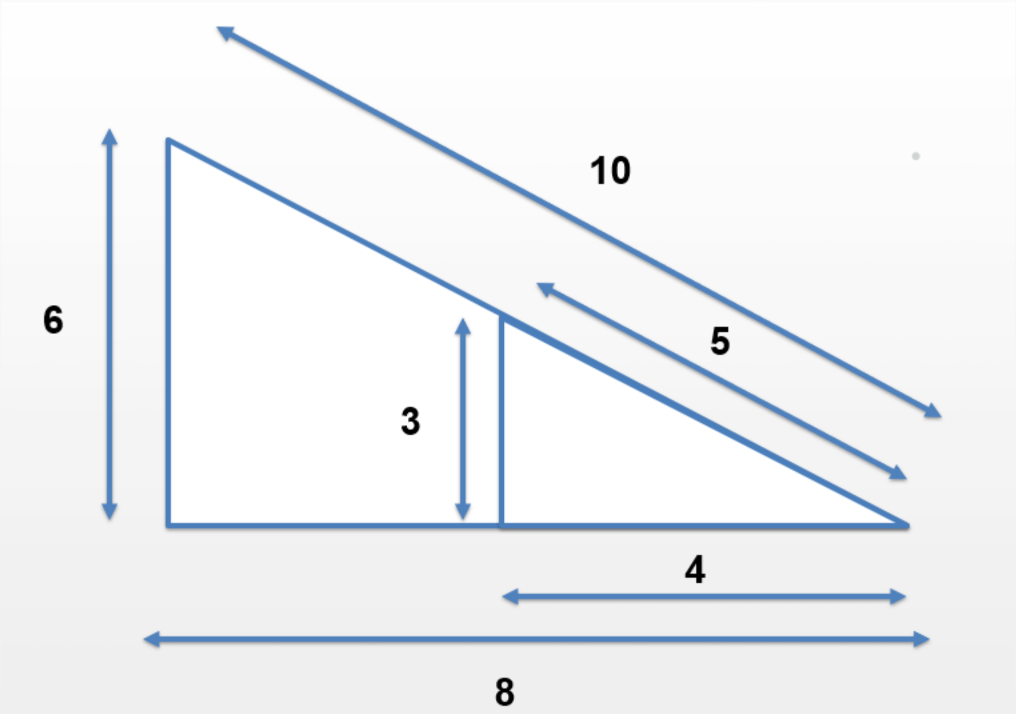
Ratio of Base to Hypotenuse

$$\frac{8}{10} = \frac{4}{5} \ldots = 0.80$$

Ratio of Height to Hypotenuse

$$\frac{6}{10} = \frac{3}{5} \ldots = 0.60$$

Similar Triangles



Ratio of Corresponding Sides

$$\frac{3}{6} = \frac{5}{10} = \frac{4}{8} \ldots = \frac{1}{2}$$

Ratio of Base to Hypotenuse

$$\frac{8}{10} = \frac{4}{5} \ldots = 0.80$$

Ratio of Height to Hypotenuse

$$\frac{6}{10} = \frac{3}{5} \ldots = 0.60$$

Ratio of Height to Base

$$\frac{6}{8} = \frac{3}{4} \ldots = 0.75$$

- In a right-angled triangle, the ratio of the lengths of sides to the angle are the same regardless of the length of the side.
- We have fixed ratios that we use to calculate

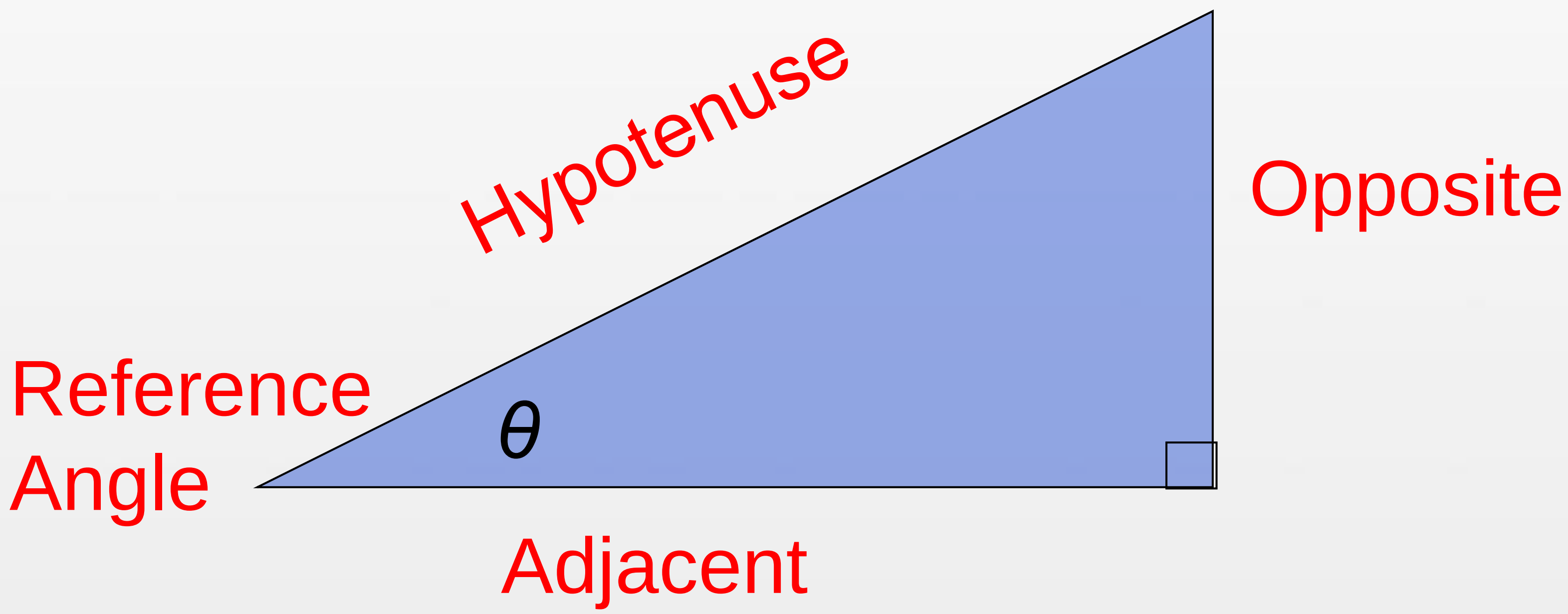
i) The size of the angle

ii) The length of the side

- These ratios are known sin, cos and tan.

Trigonometric Ratios

- We can see that there is a relationship between the trigonometric functions (Sin/Cos/Tan) and the ratios of two of the sides of a right angle triangle.

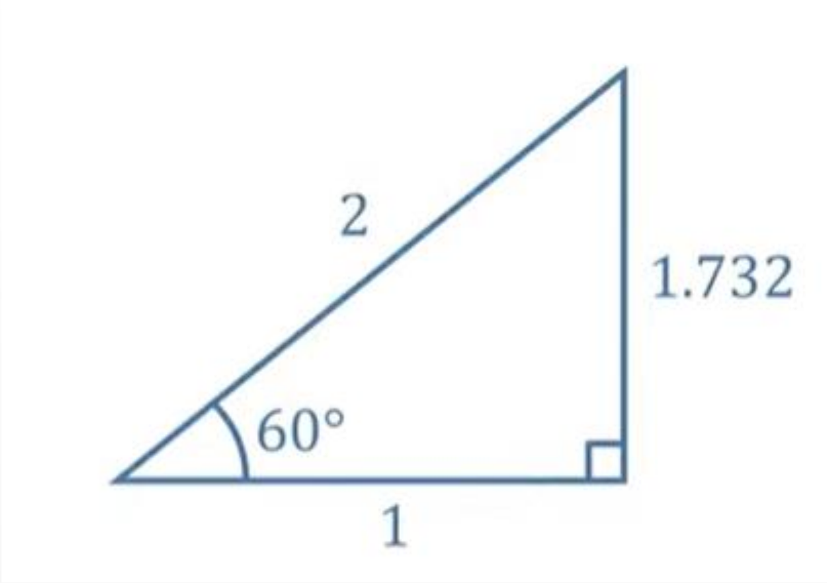


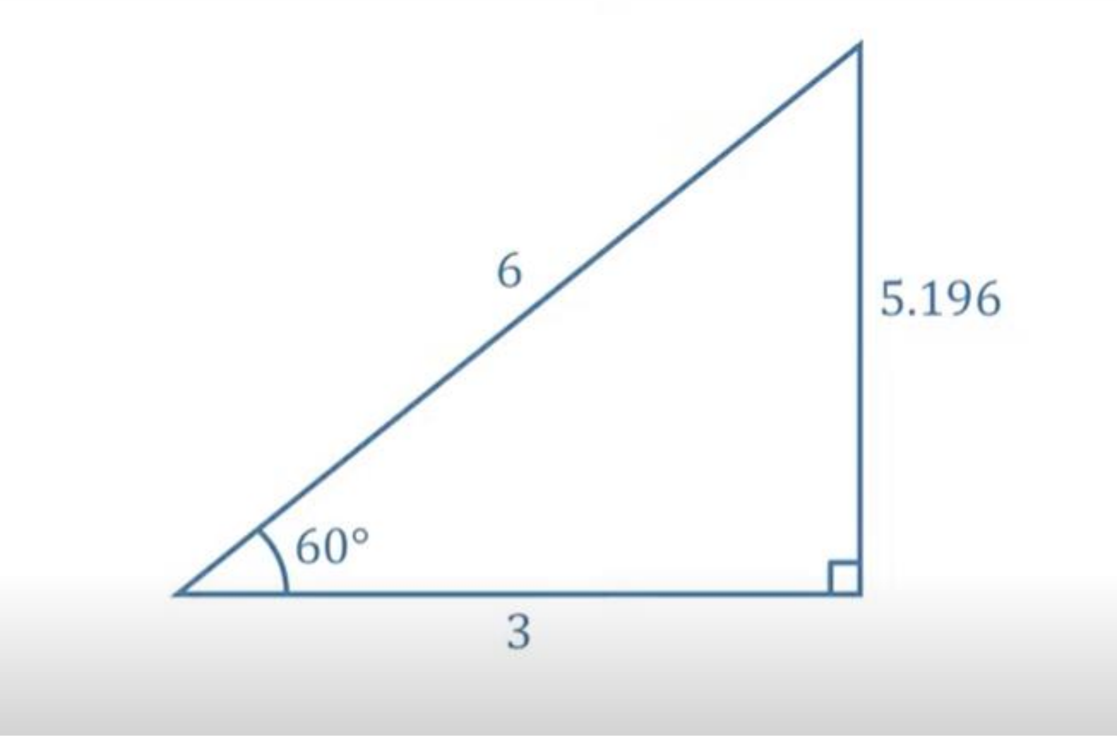
$$\text{Sin}\,\theta = \frac{Opposite}{Hypotenuse}$$

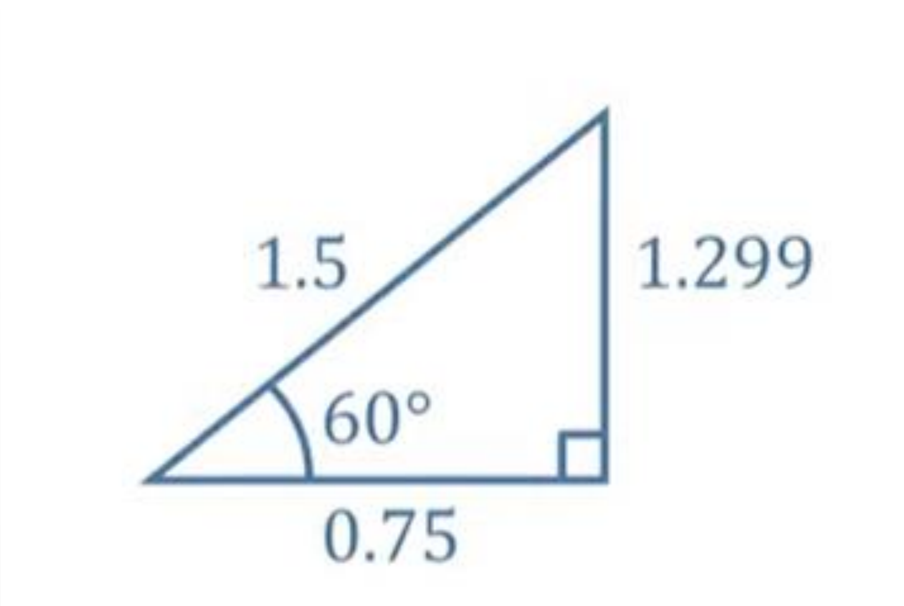
$$\text{Cos}\,\theta = \frac{Adjacent}{Hypotenuse}$$

$$\text{Tan}\,\theta = \frac{Opposite}{Adjacent}$$

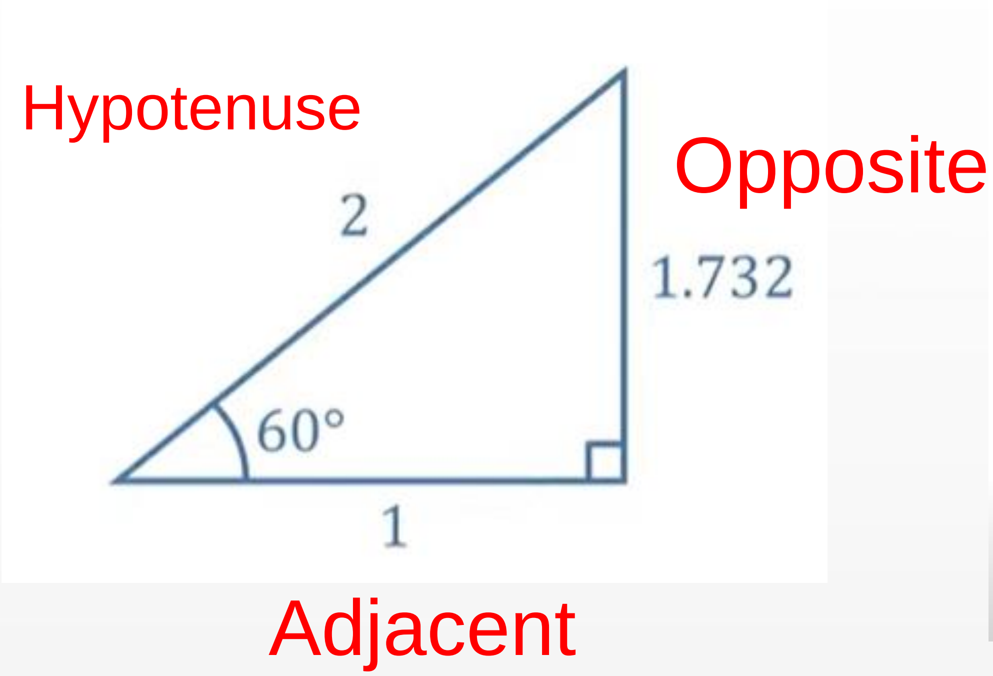
$Sin\ 60 = 0.866\ ...$

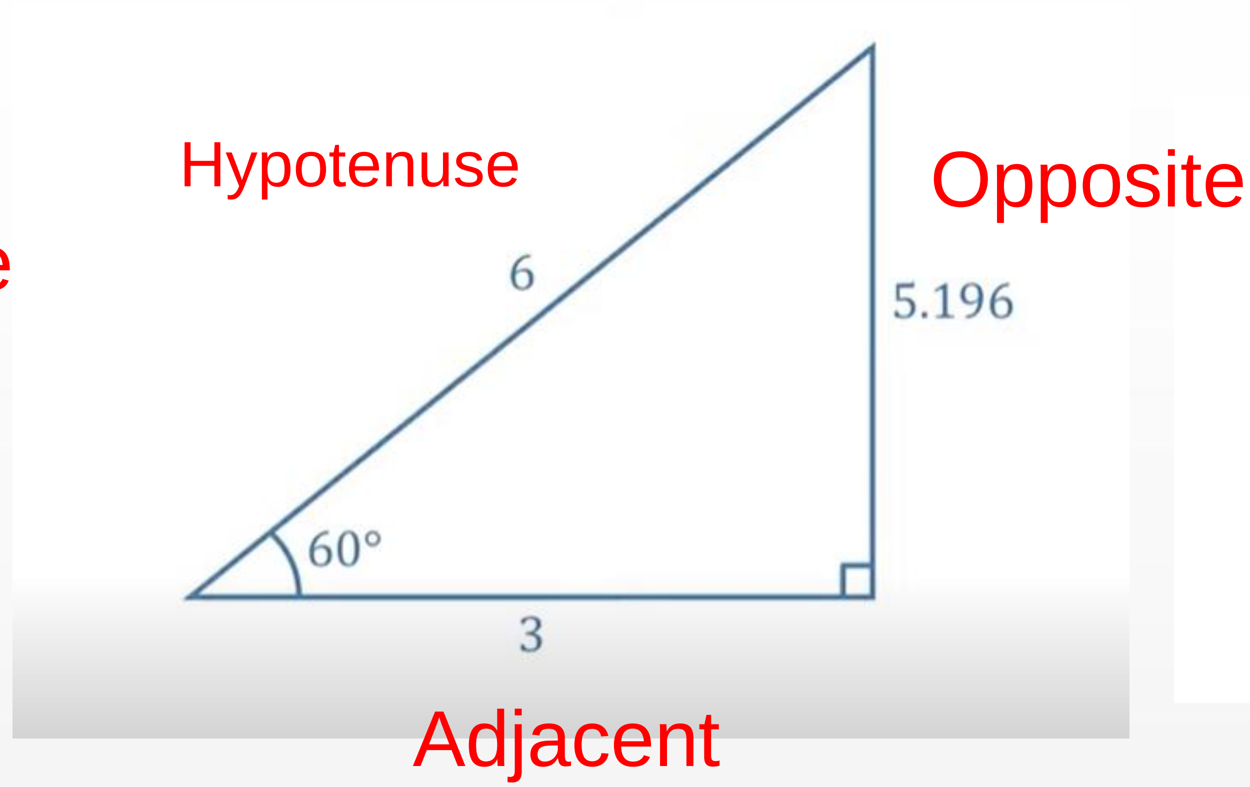


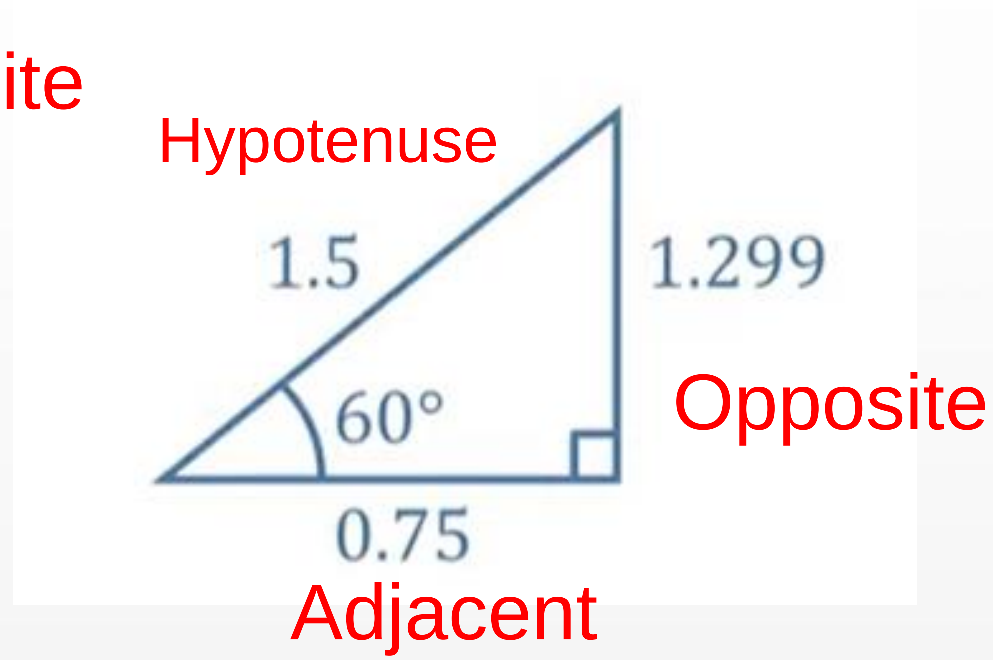




$Sin\ 60 = 0.866 \ldots$







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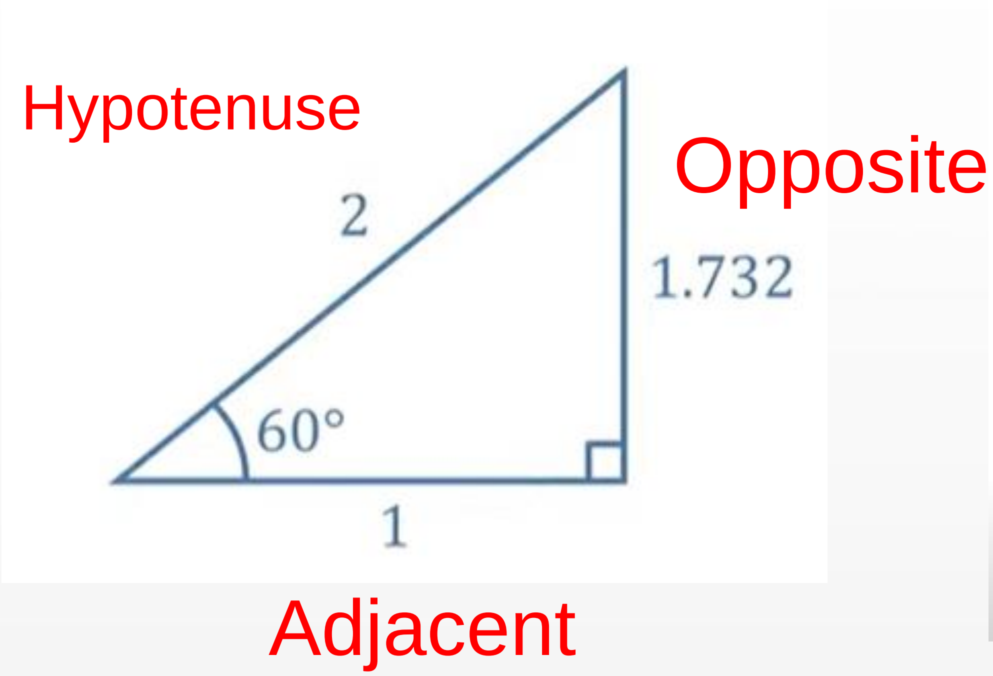
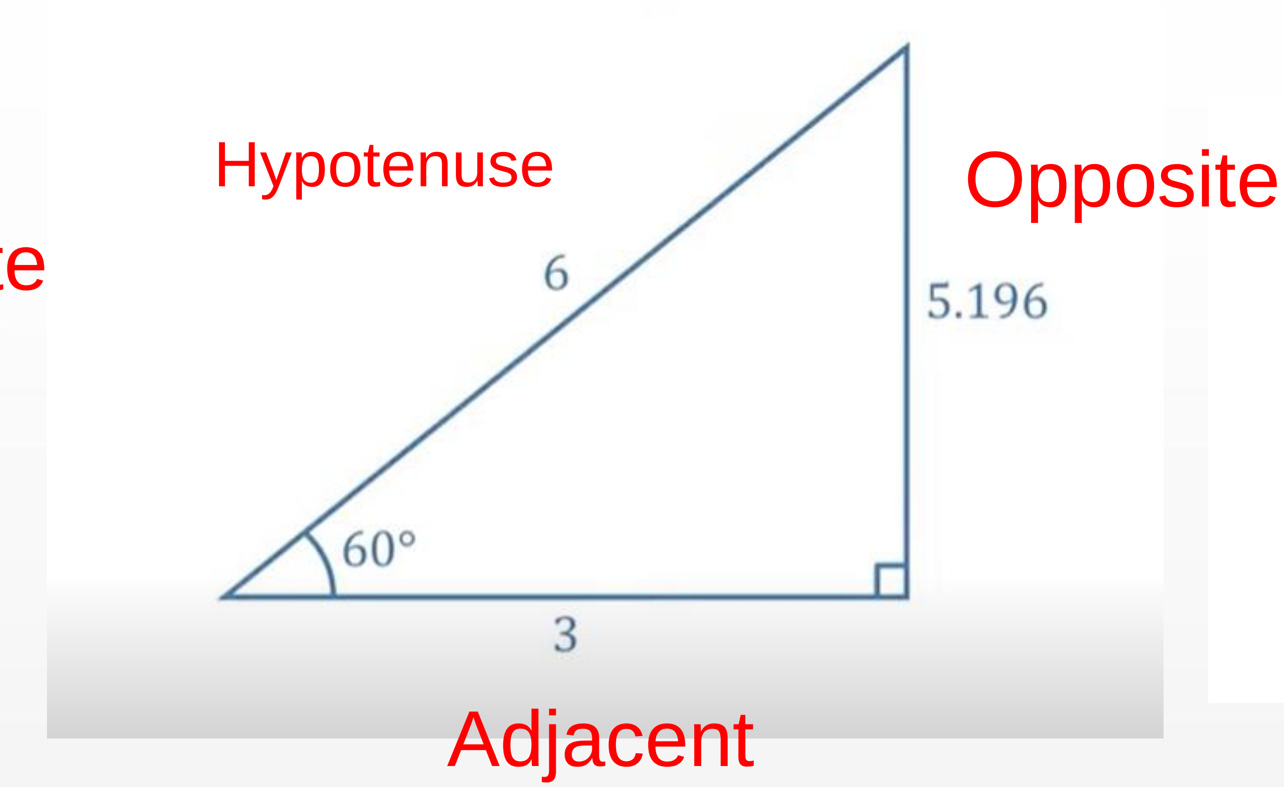
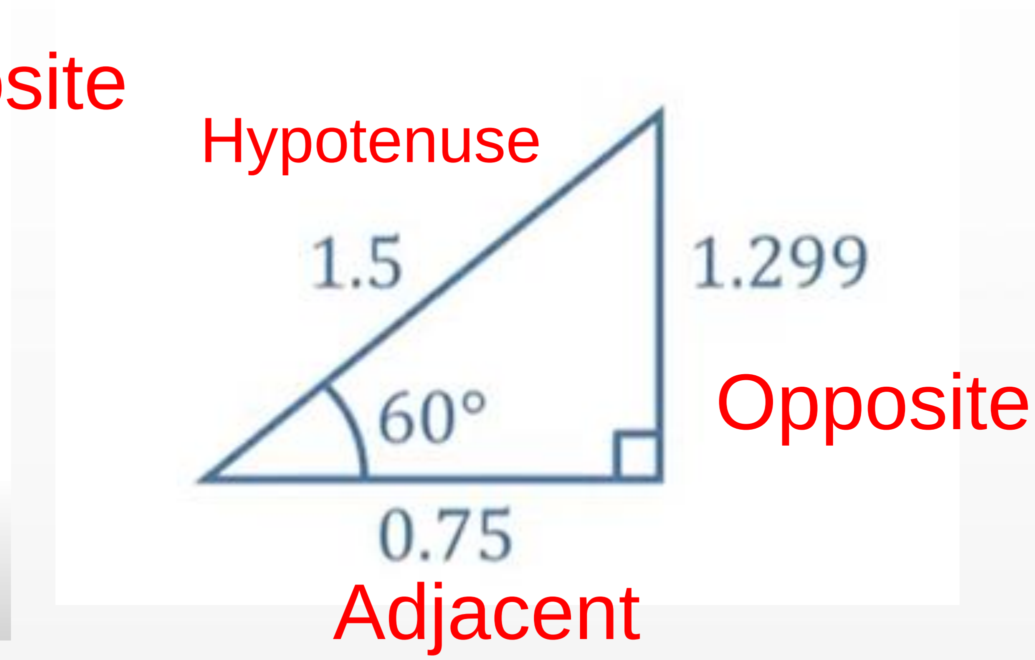




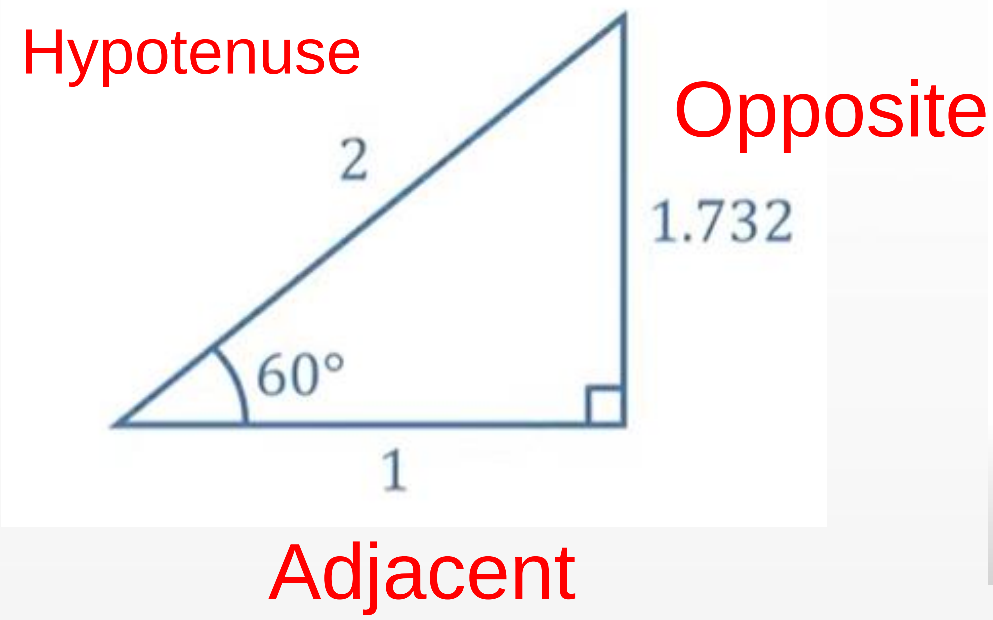


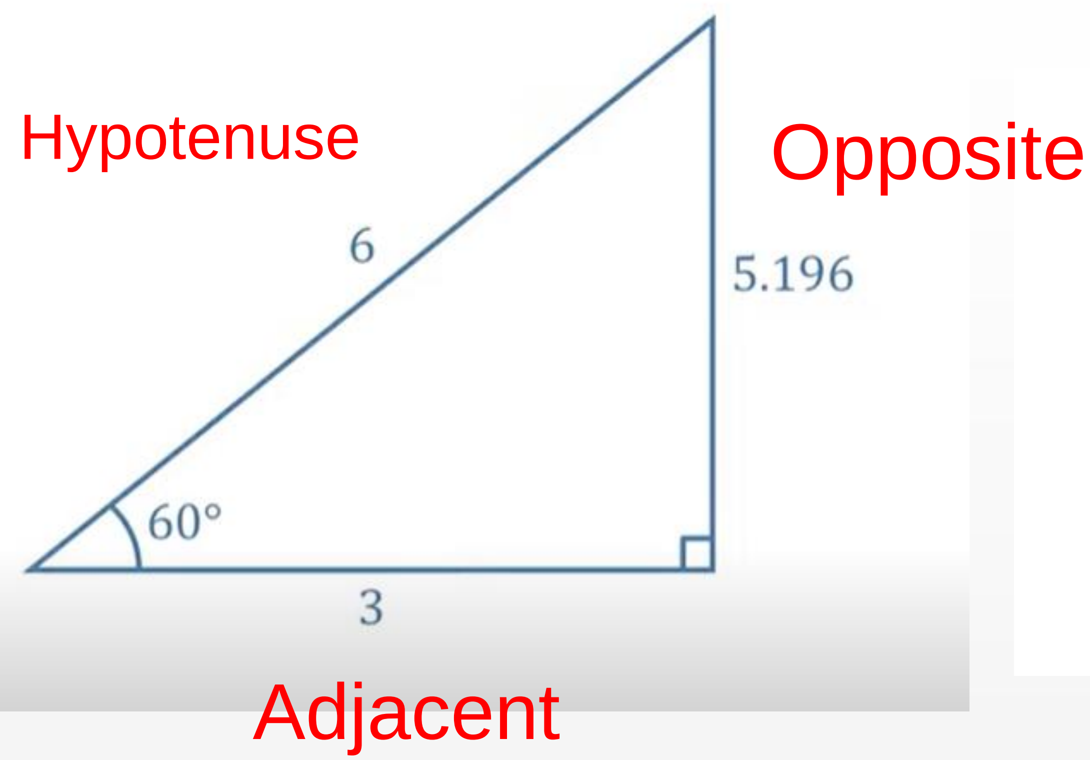
$$\frac{Opposite}{Hypotenuse}$$

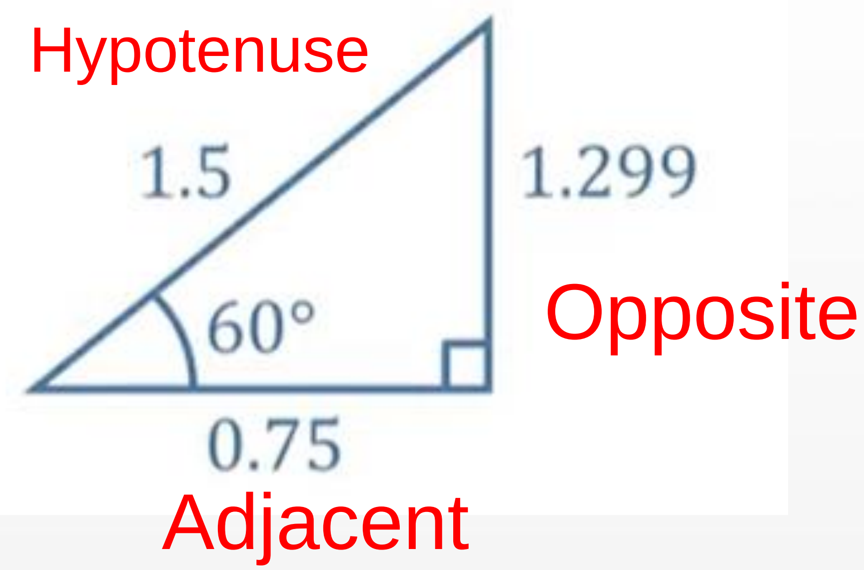
$$\frac{1.732}{2}$$

$$0.866$$

$Sin\ 60 = 0.866\ ...$







$$\frac{Opposite}{Hypotenuse}$$

$$\frac{1.732}{2}$$

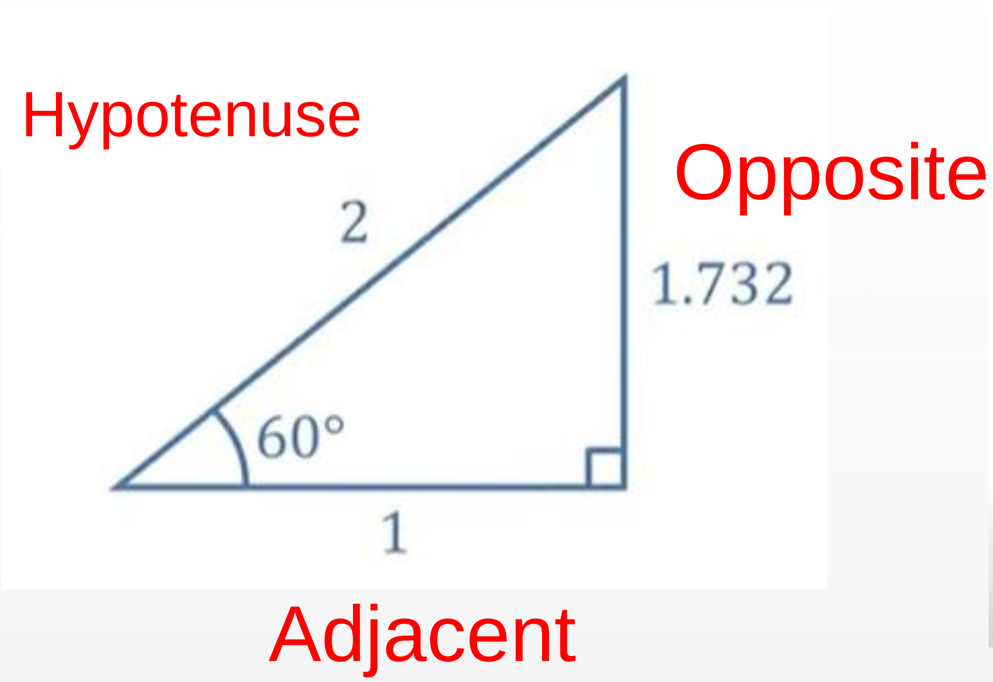
$$0.866$$

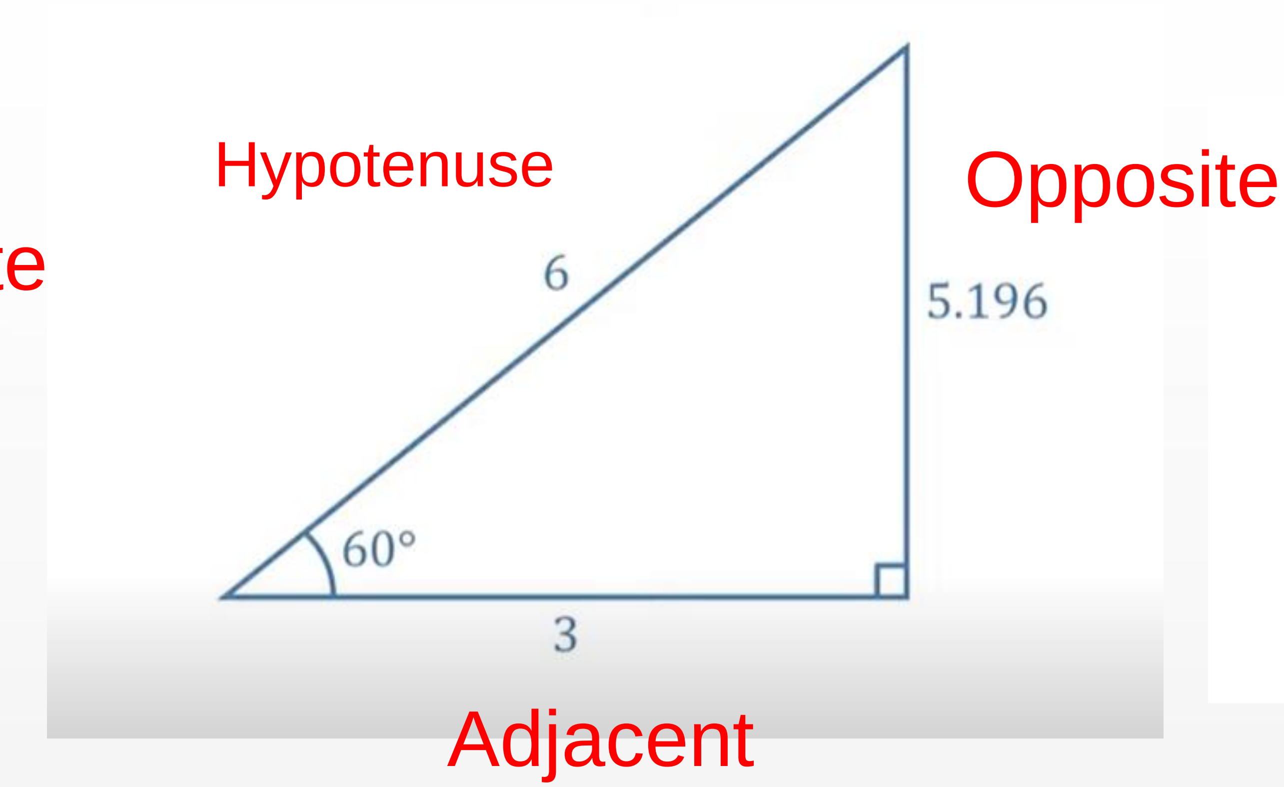
$$\frac{Opposite}{Hypotenuse}$$

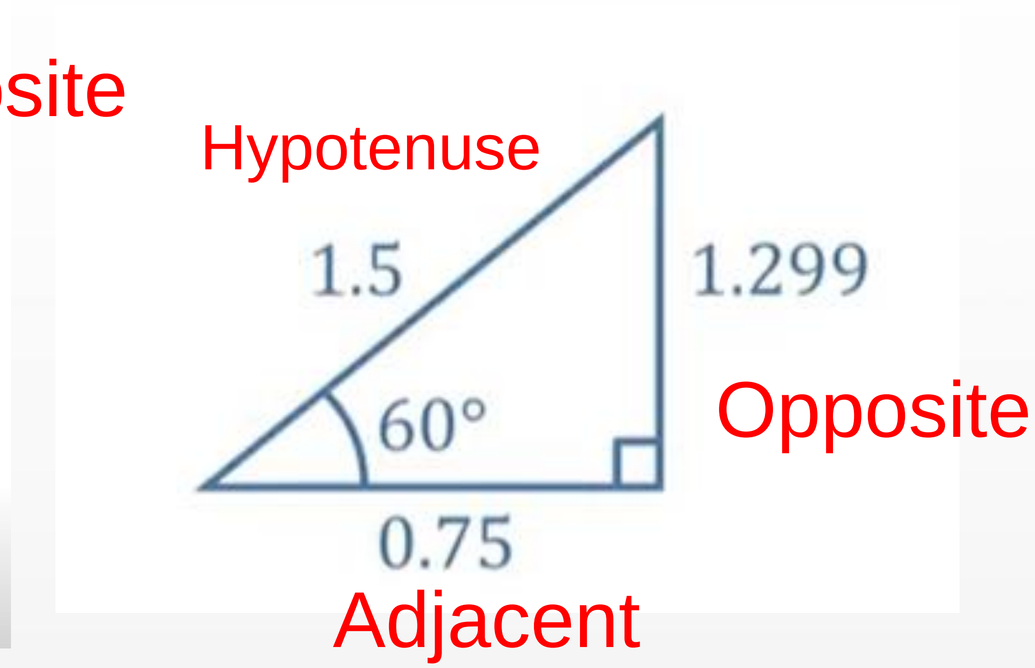
$$\frac{5.196}{6}$$

$$0.866$$

$Sin\ 60 = 0.866 \ldots$







$$\frac{Opposite}{Hypotenuse}$$

$$\frac{1.732}{2}$$

0.866

$$\frac{Opposite}{Hypotenuse}$$

$$\frac{5.196}{6}$$

0.866

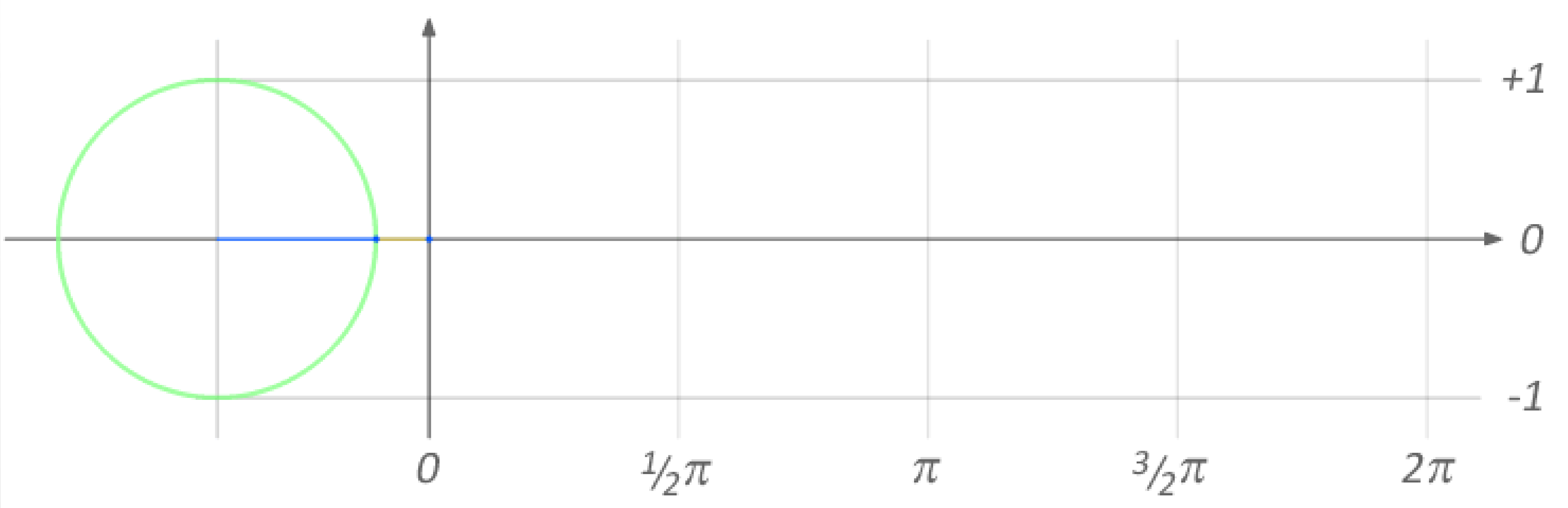
$$\frac{Opposite}{Hypotenuse}$$

$$\frac{1.299}{1.5}$$

0.866

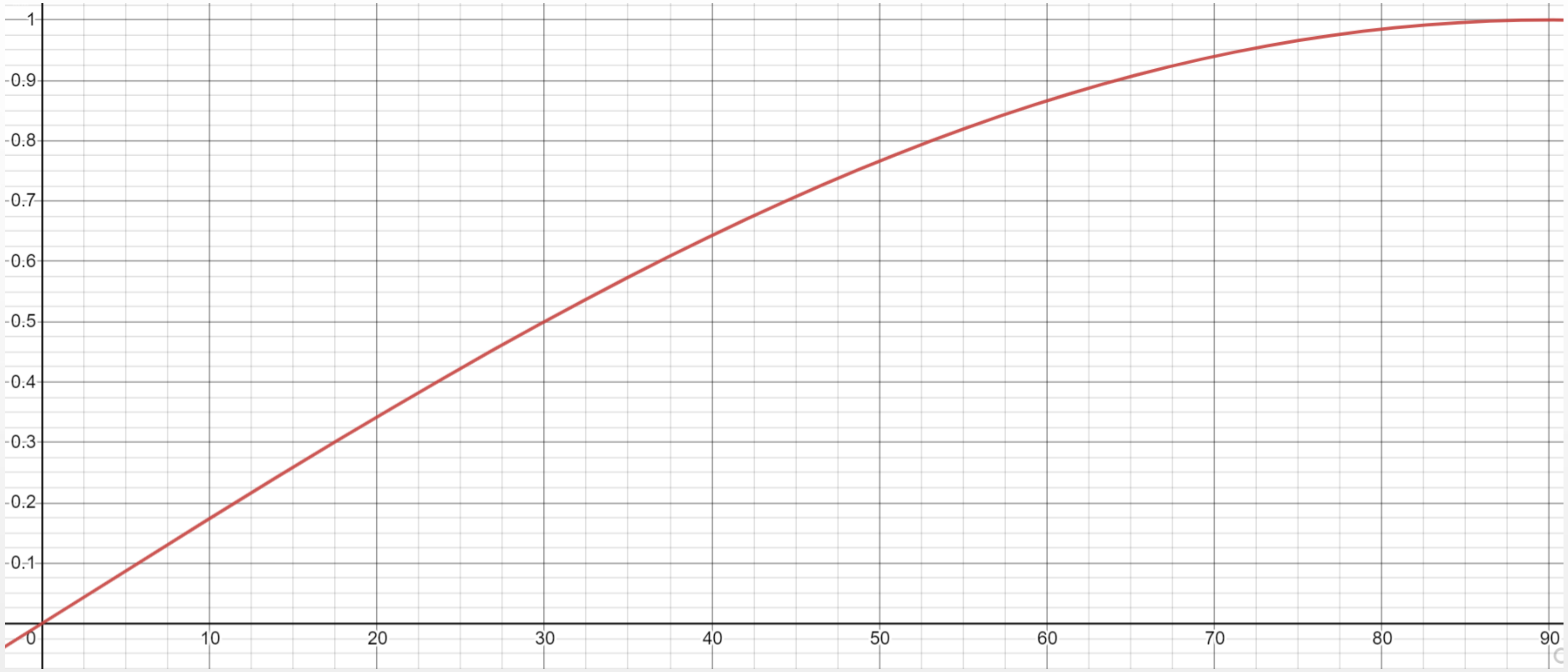
- Therefore, Sin of the angle is always equal to the ratio of the opposite and hypotenuse sides ($Sin\theta = \frac{O}{H}$)
- In a similar way, Cos of an angle is always equal to the ratio of the adjacent and hypotenuse sides ($Cos\theta = \frac{A}{H}$) and Tan of an angle is equal to the ratio of Sin of the angle and Cos of the angle ($Tan\theta = \frac{Sin\theta}{Cos\theta} = \frac{O}{A}$)
- These functions and their ratios can be used to find missing angles and sides from various right angled triangles.

Sine Function



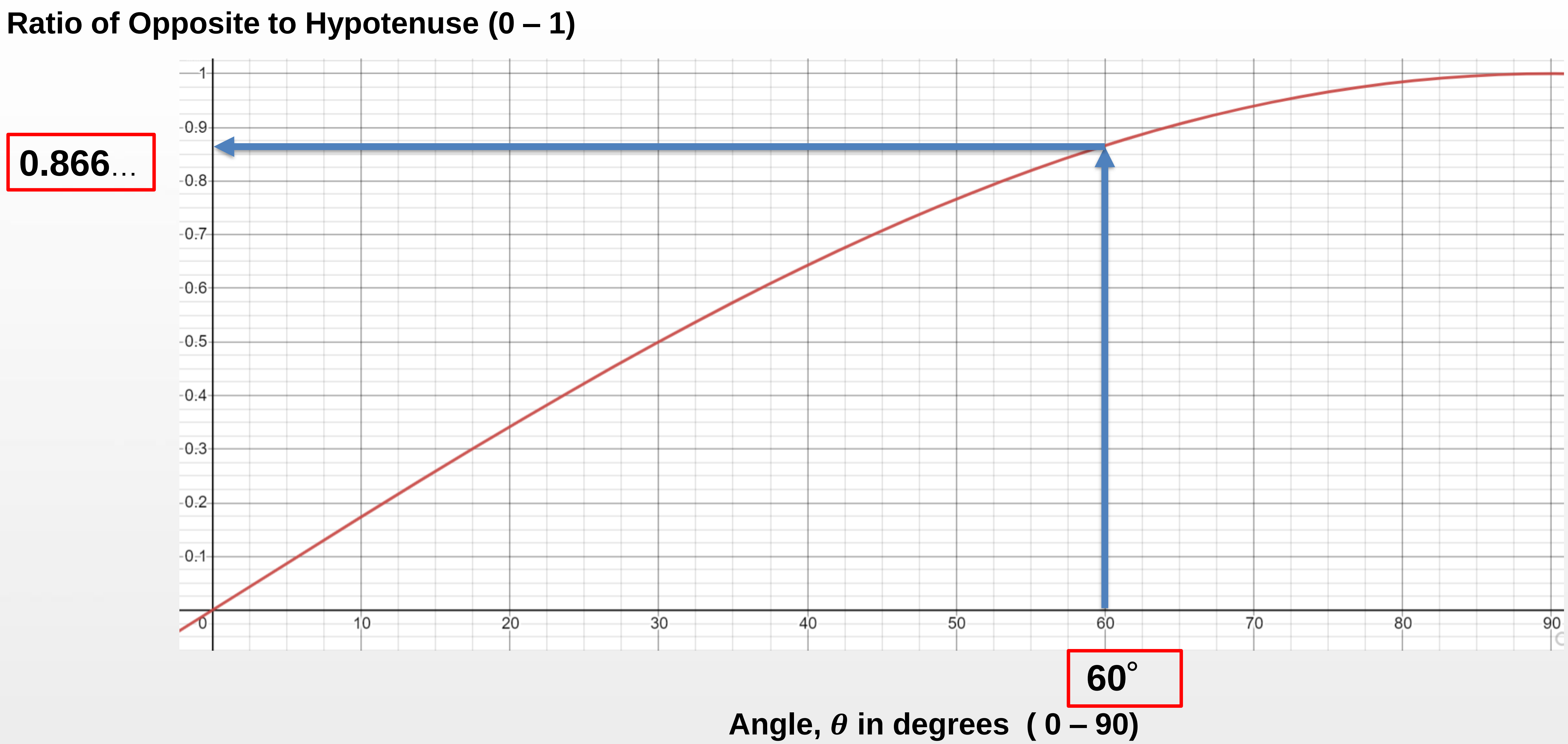
$$Sin\ 60 = 0.866 \ldots$$

Ratio of Opposite to Hypotenuse (0 – 1)



Angle, θ in degrees (0 – 90)

$$Sin\ 60 = 0.866\ ...$$

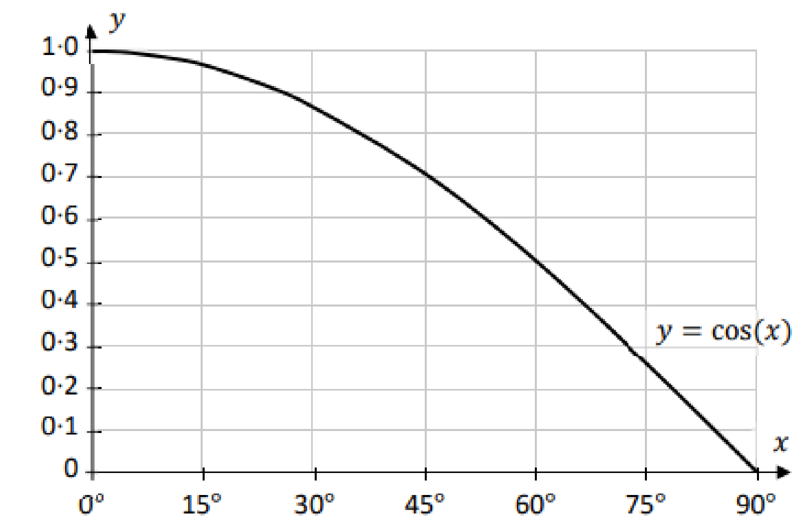


Ratio of Opposite to Hypotenuse (0 – 1)
0.866…
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
10
20
30
40
50
60
70
80
90
60°
Angle, θ in degrees (0 – 90)

Sin/Cos/ Tan

- Questions surrounding Sin/Cos/ Tan has appeared on recent SEC Junior Cycle Sample Papers – underlining the importance of student understanding of this concept

The co-ordinate diagram below shows the graph of the function $y = \cos(x)$, for $0° \le x \le 90°$.



(a) Use a calculator to work out the value of $\sin(60°)$, correct to one decimal place.

(b) **Draw** the graph of $y = \sin(x)$ on the diagram above, using the same axes, scales, and domain. Note that $\sin(0°) = 0$ and $\sin(90°) = 1$.

Reflection:

- How did you approach teaching Sin/Cos/Tan before?
- How might your practice change as a result of this video?
- Is there anything from this video that you would like to learn more about?

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References:

- https://commons.wikimedia.org/wiki/File:Sine_curve_drawing_animation.gif

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Trigonometry

Teacher CPD #4: Trigonometric Functions

Relationship between Trigonometric Functions and the Inverse

Inverse Operations

Inverse Operations	
+	−
÷	x
$(\ )^2$	$\sqrt{\ }$

Inverse Operations are pairs of mathematical manipulations in which one operation undoes the action of the other.

What is $sin^{-1}x$?

- We know that Sin/Cos/Tan of an angle correspond to a ratio of the sides of the triangle.

$$Sin(Angle) = \frac{Opposite}{Hypotenuse}$$

$$(Angle) = Sin^{-1}\left(\frac{Opposite}{Hypotenuse}\right)$$

What is $sin^{-1}x$?

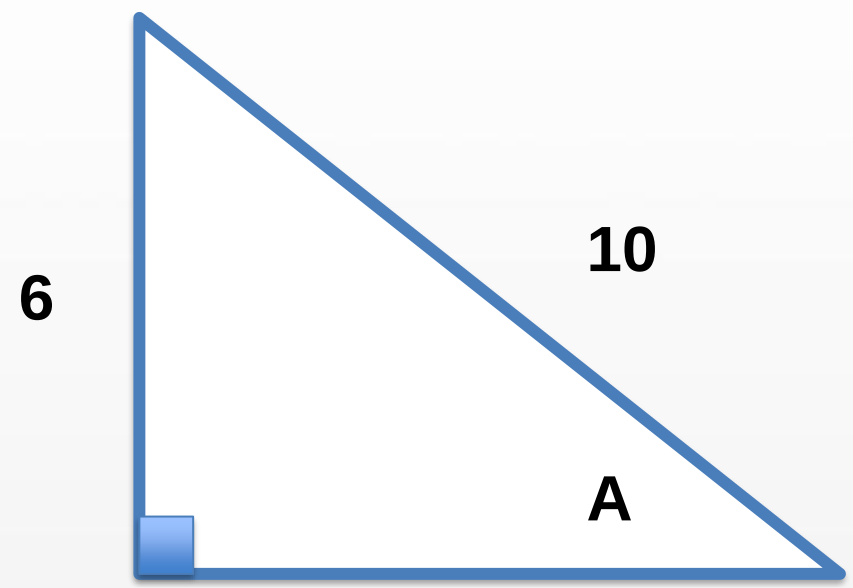
- We know that Sin/Cos/Tan of an angle correspond to a ratio of the sides of the triangle.

$$Sin(Angle) = \frac{Opposite}{Hypotenuse}$$

$$(Angle) = Sin^{-1}\left(\frac{Opposite}{Hypotenuse}\right)$$

It answers the question… What angle has a Sin of $\frac{Opposite}{Hypotenuse}$

Inverse Sine Function



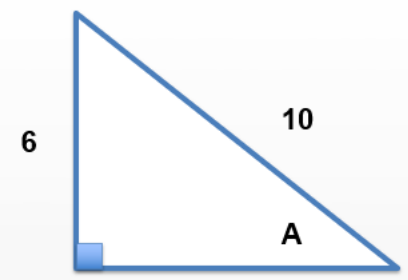
$$SinA = \frac{O}{H}$$

$$Sin\ A = \frac{6}{10}$$

$$A = Sin^{-1}(\frac{6}{10})$$

$$A = 36.87°$$

Sine Function



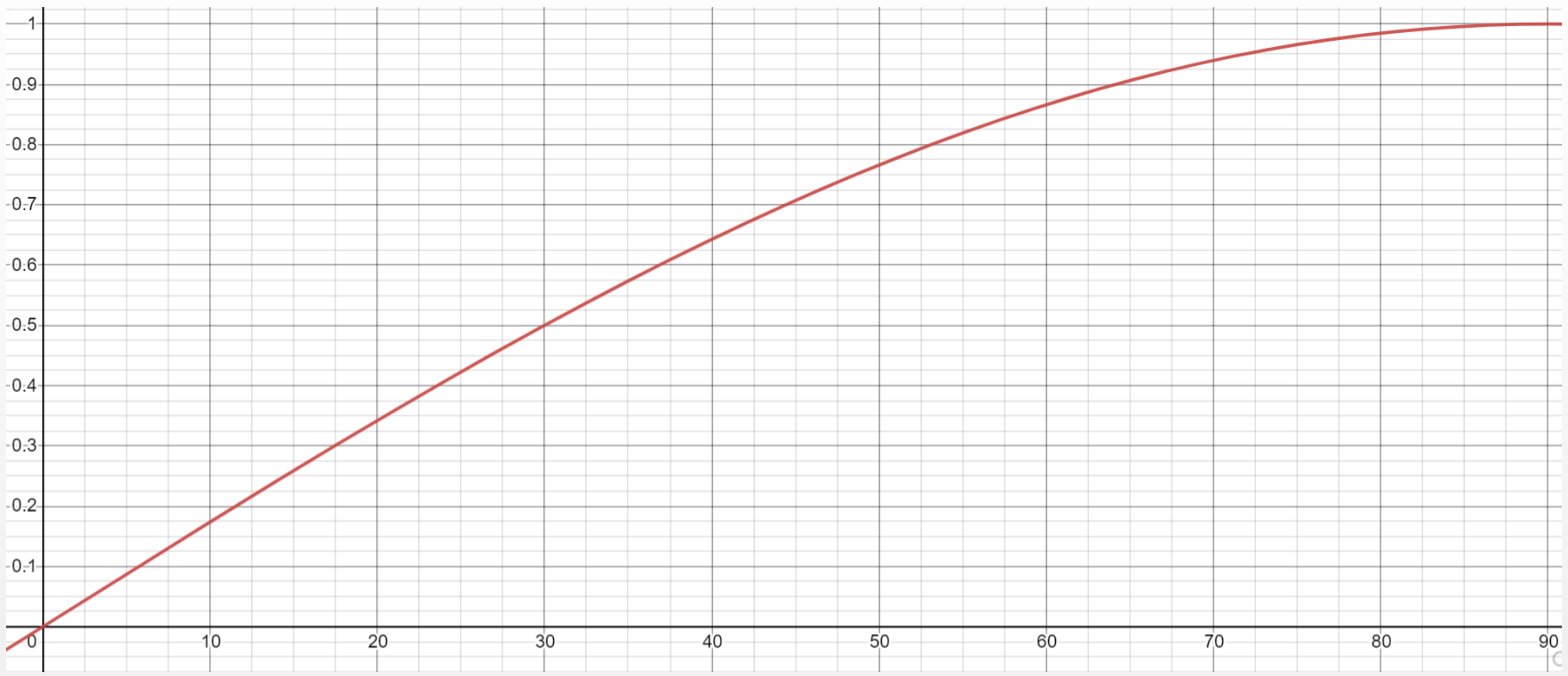
$$Sin A = \frac{O}{H}$$

$$Sin\ A = \frac{6}{10}$$

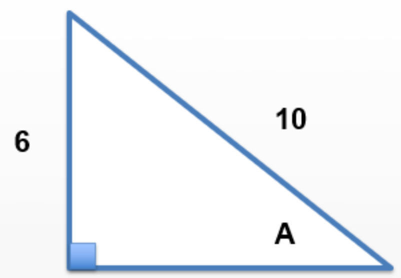
$$A = Sin^{-1}(\frac{6}{10})$$

$$A = 36.87°$$

$$Sin\ A = \frac{6}{10}$$



Sine Function



$$Sin\,A = \frac{O}{H}$$

$$Sin\,A = \frac{6}{10}$$

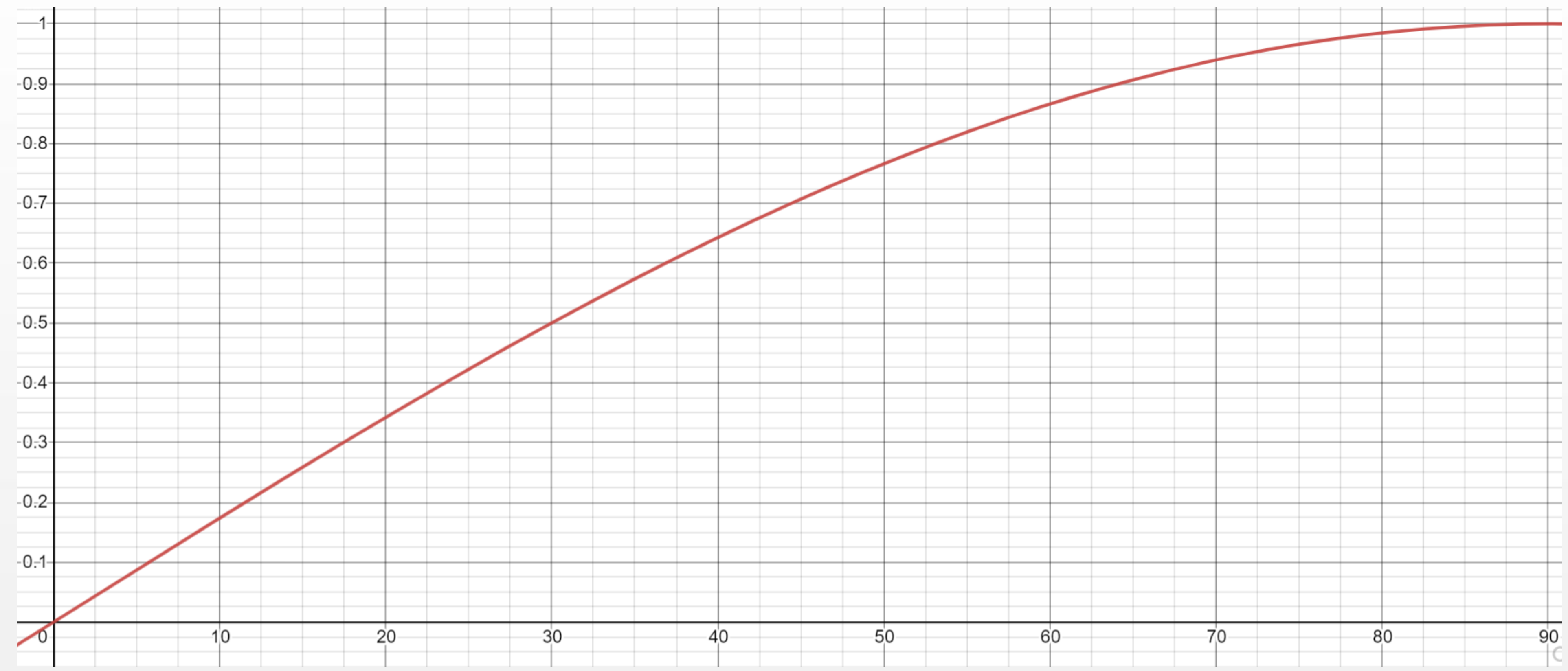
$$A = Sin^{-1}(\frac{6}{10})$$

$$A = 36.87°$$

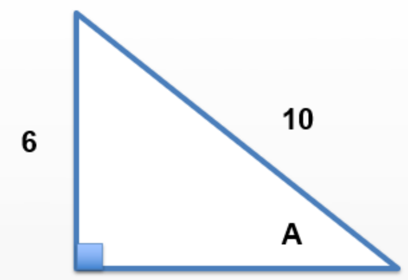
Output

$$Sin\,A = \frac{6}{10}$$

Input



Sine Function



$$Sin\,A = \frac{O}{H}$$

$$Sin\,A = \frac{6}{10}$$

$$A = Sin^{-1}(\frac{6}{10})$$

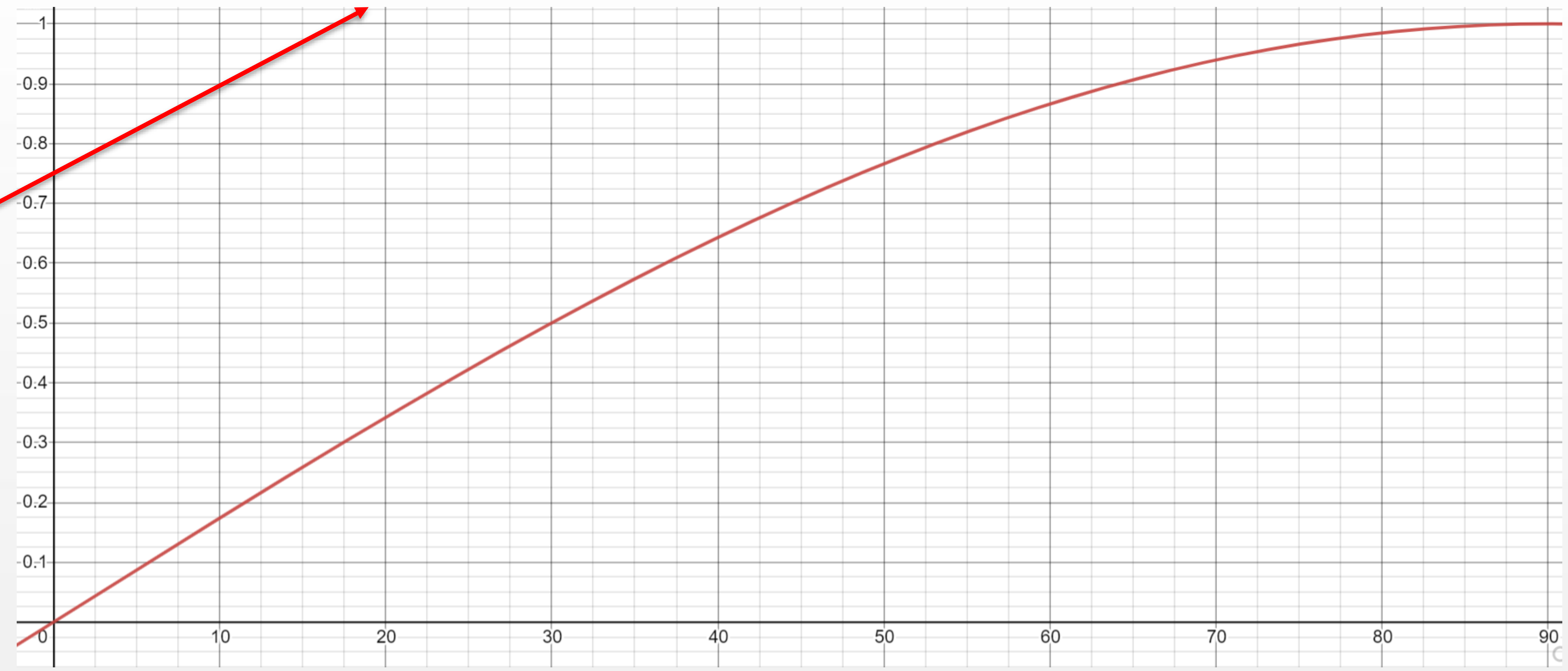
$$A = 36.87°$$

Output

$$Sin\,A = \frac{6}{10}$$

Input

Ratio of Opposite to Hypotenuse (0 – 1)



Angle (0 – 90)

Inverse Trigonometric Functions

Trigonometric functions input angles and output side ratios		Inverse trigonometric functions input side ratios and output angles
$\sin(\theta) = \frac{\text{opposite}}{\text{hypotenuse}}$	$\rightarrow$	$\sin^{-1}\left(\frac{\text{opposite}}{\text{hypotenuse}}\right) = \theta$
$\cos(\theta) = \frac{\text{adjacent}}{\text{hypotenuse}}$	$\rightarrow$	$\cos^{-1}\left(\frac{\text{adjacent}}{\text{hypotenuse}}\right) = \theta$
$\tan(\theta) = \frac{\text{opposite}}{\text{adjacent}}$	$\rightarrow$	$\tan^{-1}\left(\frac{\text{opposite}}{\text{adjacent}}\right) = \theta$

Inverse Sine Function

- For a function to have an inverse function, it must be one-to-one (bijective)—that is, it must pass the Horizontal Line Test exactly once.
- $y = \sin x$ does not pass the test because different values of x have the same y-value.

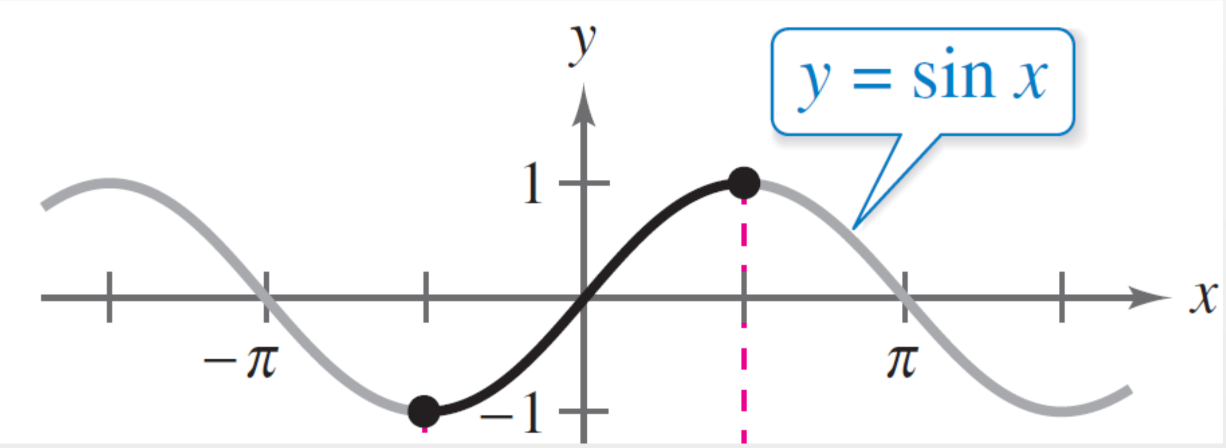
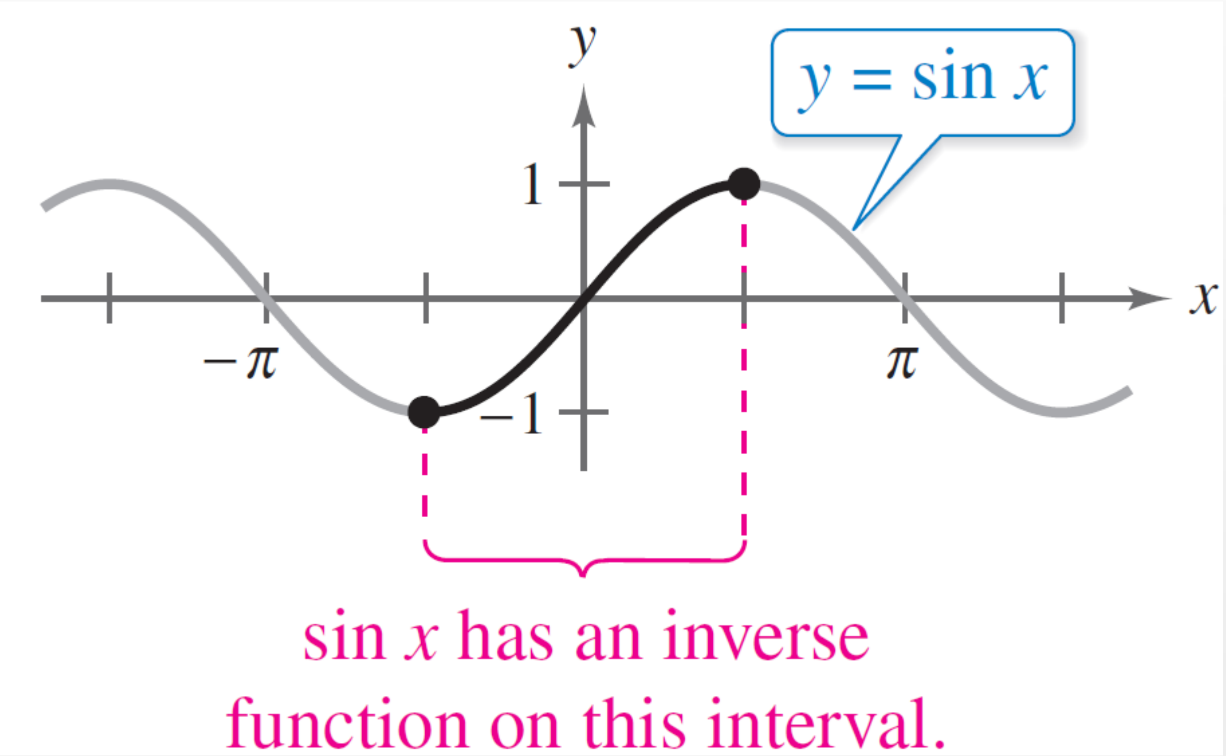


Figure 4.67

Inverse Sine Function

- However, when you restrict the domain to the interval $-\frac{\pi}{2} \leq x \leq \frac{\pi}{2}$, the function does have an inverse (passes the horizontal line test).

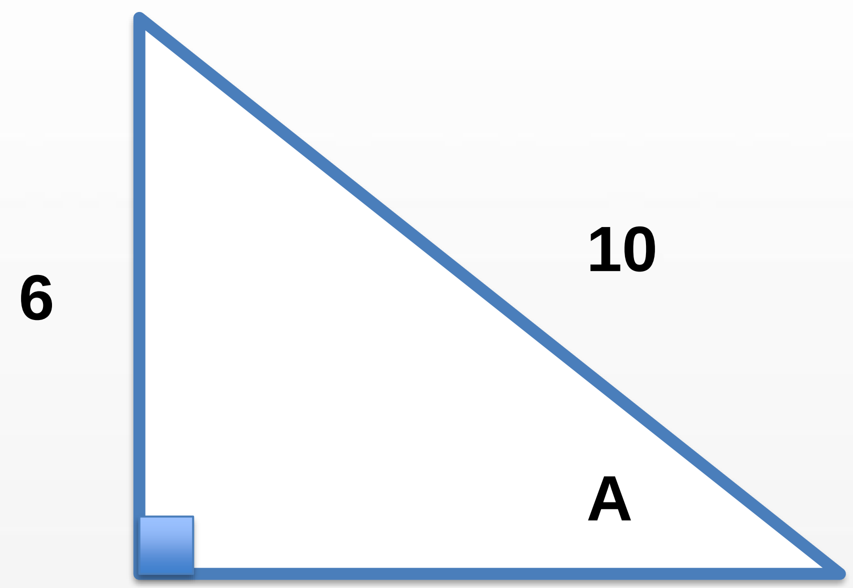


Inverse Sine Function

- So, on the restricted domain $-\frac{\pi}{2} \leq x \leq \frac{\pi}{2}$, $y = \sin x$ has a unique inverse function called the **inverse sine function.** It is denoted by

- $y = \arcsin x$ or $y = \sin^{-1} x$.

- The notation $\sin^{-1} x$ is similar to the inverse function notation $f^{-1}(x)$.



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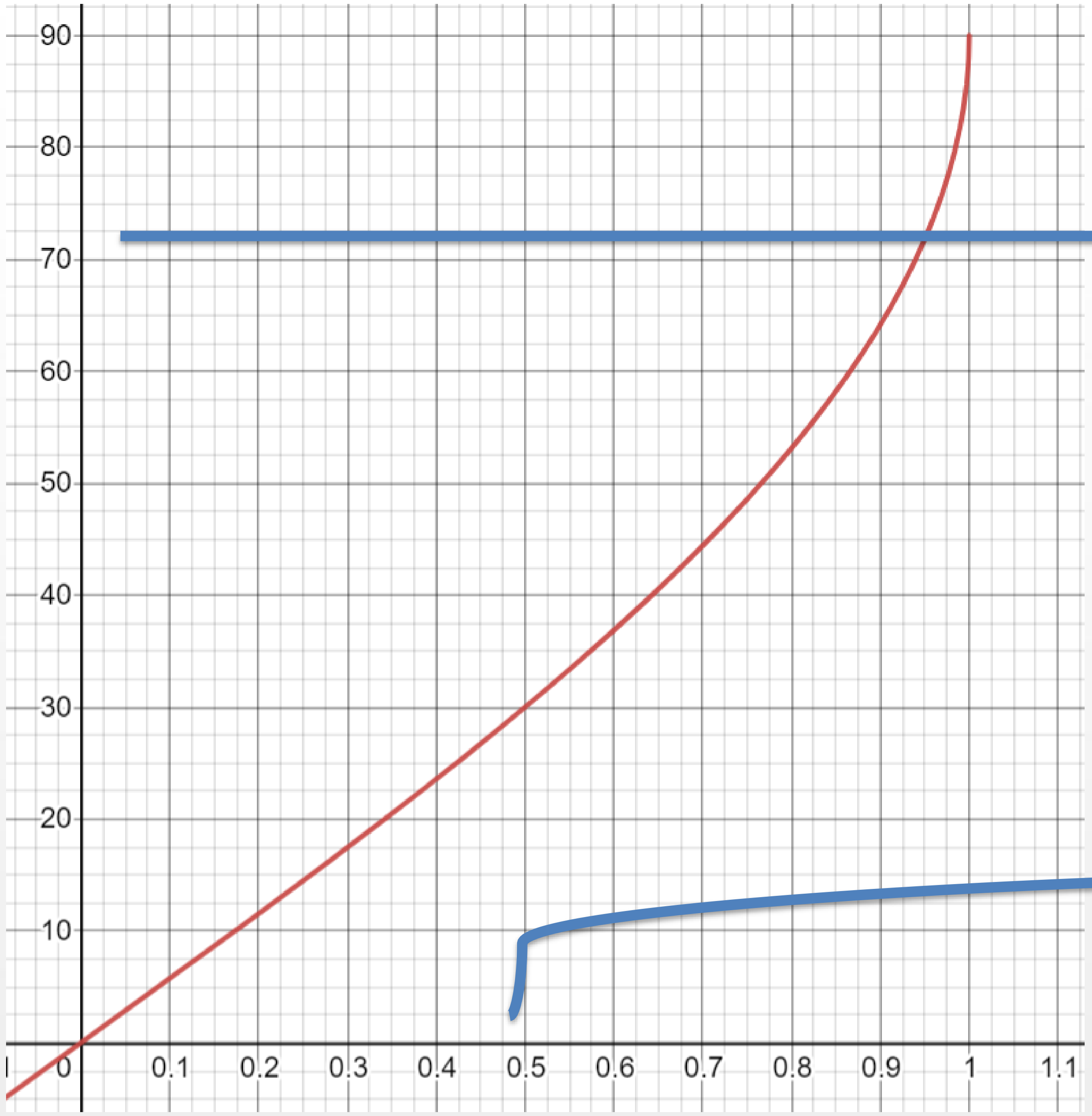
$$Sin\ A = \frac{6}{10}$$

$$A = Sin^{-1}(\frac{6}{10})$$

$$A = 36.87°$$

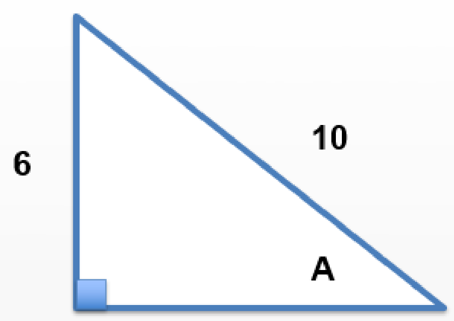
Inverse Sine Function



Angle θ (0 – 90 degrees)

Ratio of Opposite to Hypotenuse (0 – 1)

Inverse Sine Function



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$$Sin\ A = \frac{6}{10}$$

$$A = Sin^{-1}(\frac{6}{10})$$

$$A = 36.87°$$

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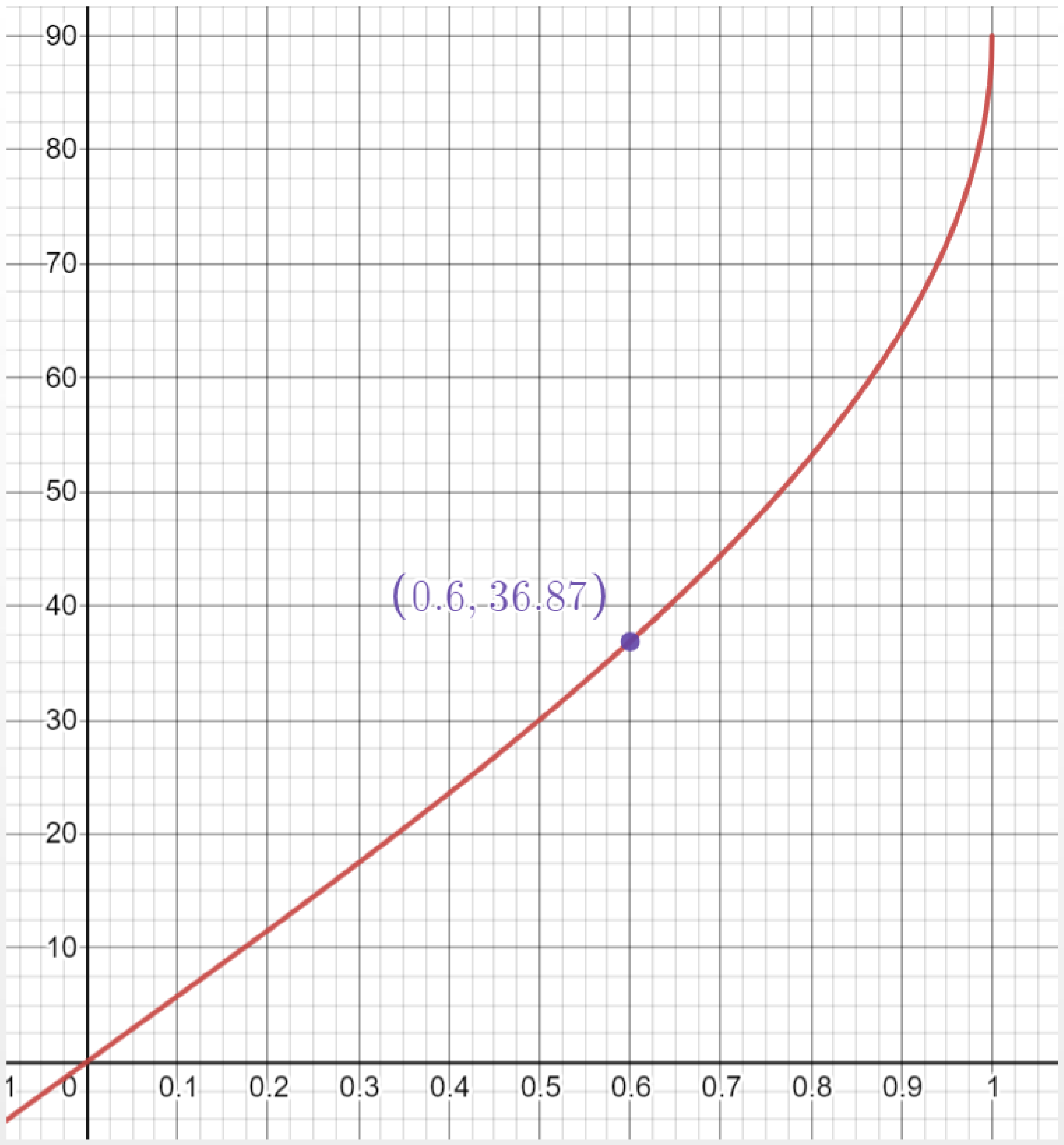
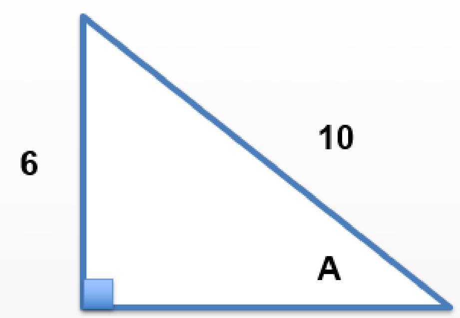


Inverse Sine Function



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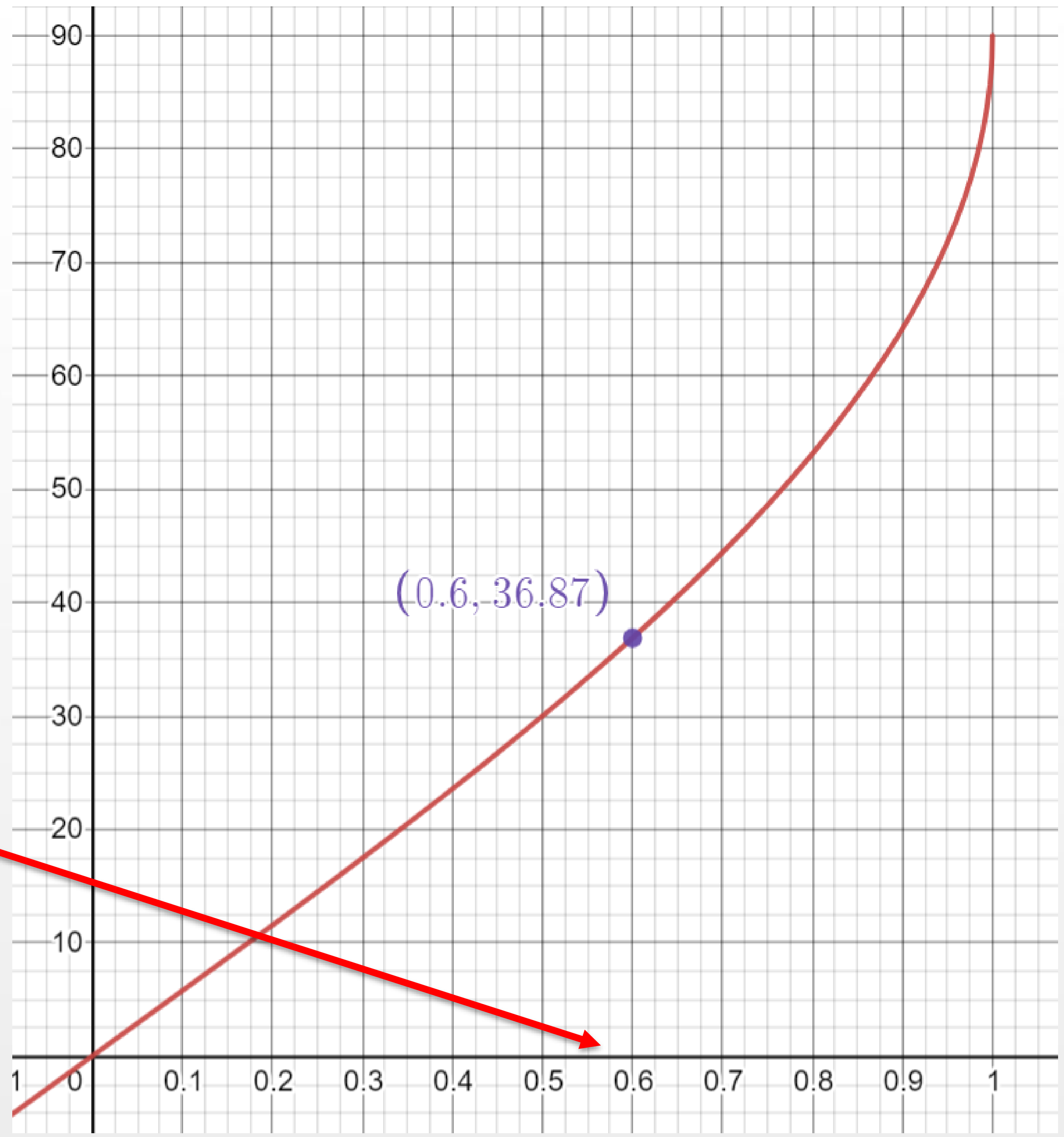
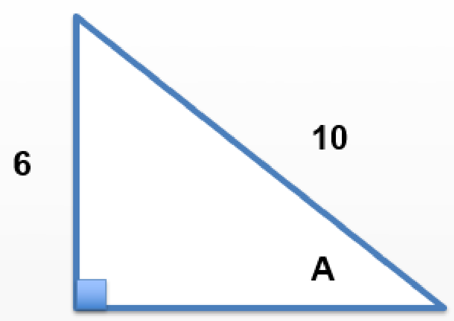
$$A = Sin^{-1}(\frac{6}{10})$$

$$A = 36.87°$$

Input



Inverse Sine Function



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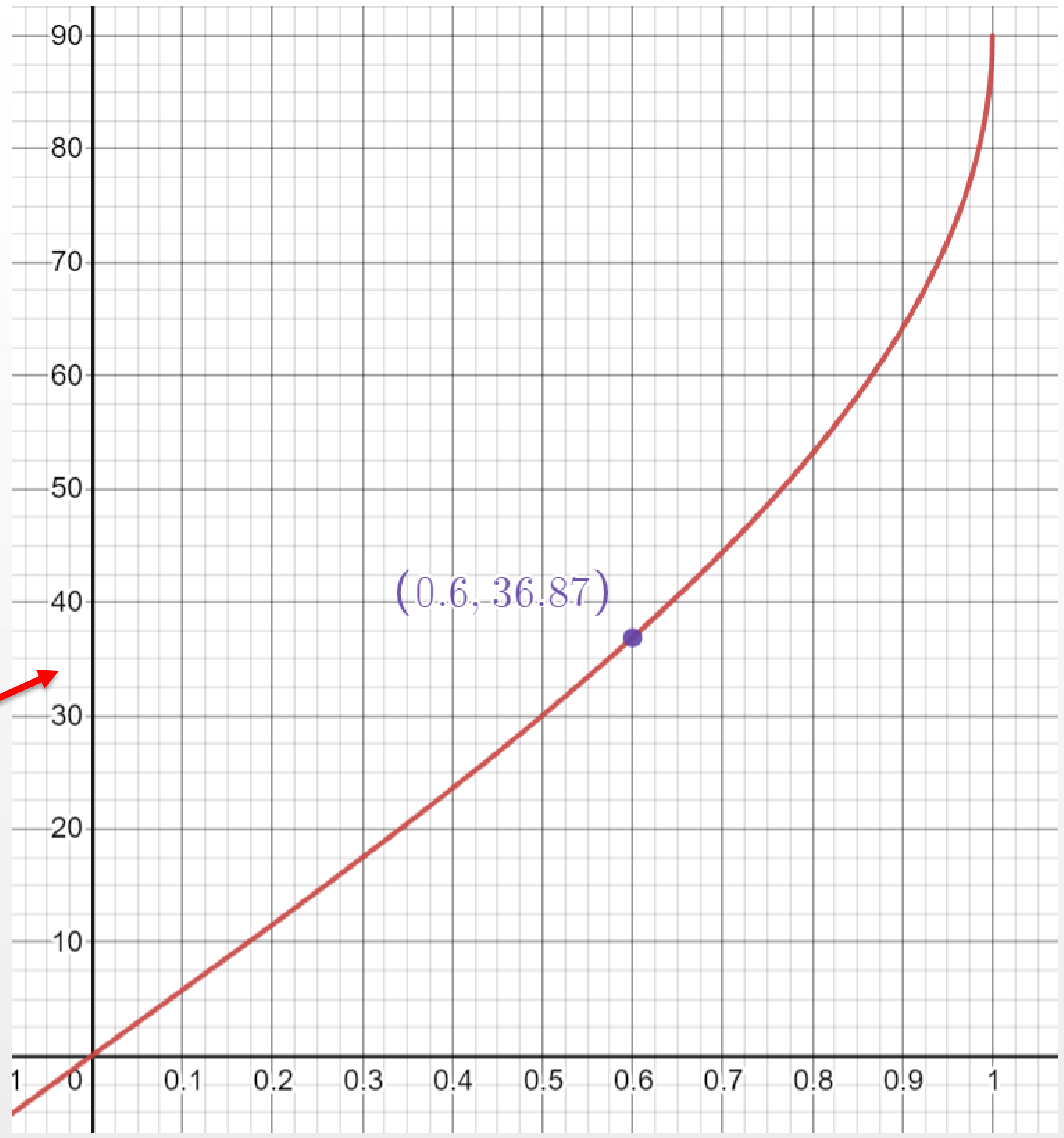
$$A = 36.87°$$

Input

Output



Inverse Trig Functions

- In a similar way, this process can also be applied to Cos and Tan functions and their respective inverses.
- As teachers, it is important for us to understand the link between the various trigonometric functions and their inverses in order for us to teach in a way that promotes conceptual understanding of the concept.
- Students will require a knowledge of trigonometric graphs and their inverses as part of the Higher Level Leaving Cert Course.

Reflection:

- How did you approach teaching Inverse Functions before?
- How might your practice change as a result of this video?
- Is there anything from this video that you would like to learn more about?

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Trigonometry

Teacher CPD #5: Classroom Based Assessments

Building Learning Experiences using the Problem Solving Cycle

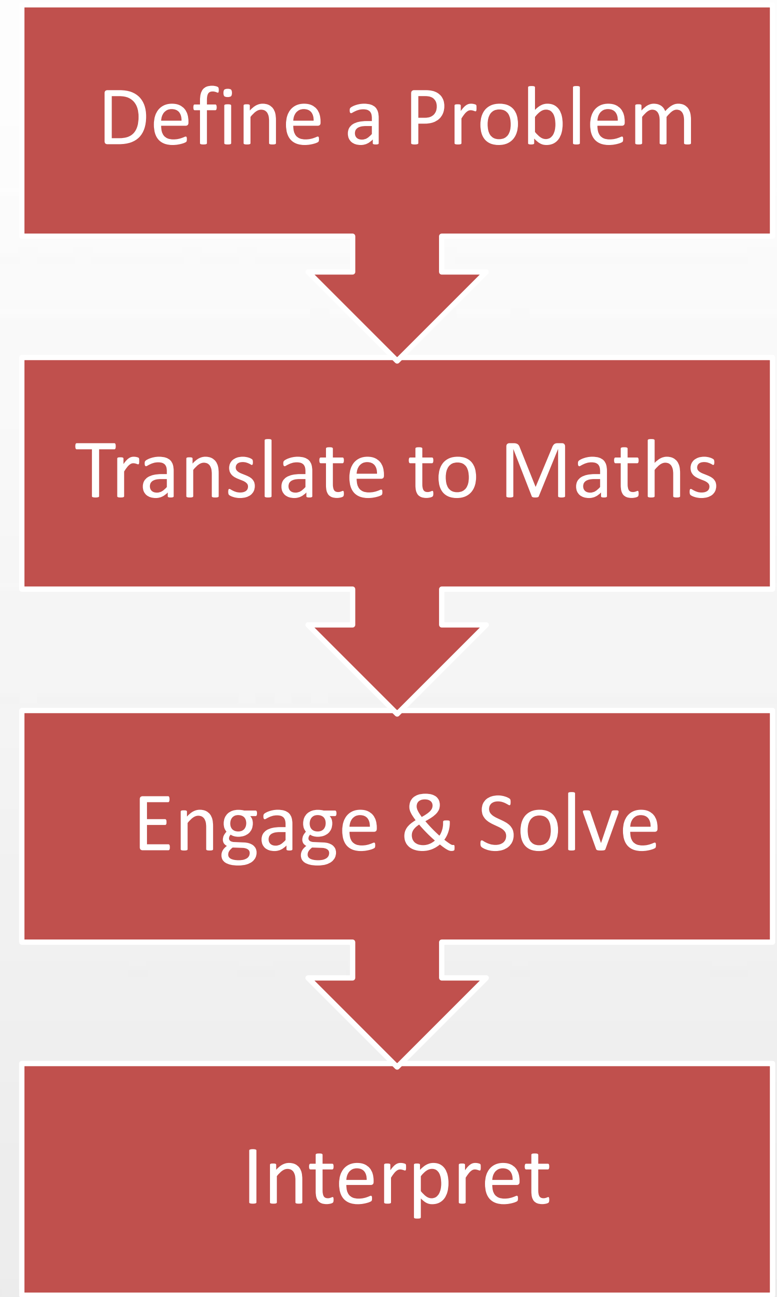
CBA 1
Mathematical Investigation

- End of 2nd Year
- Define a problem
- Decompose/simplify it into manageable parts
- Engage with the problem using mathematical strategies.
- Interpret any findings

1. Teach Content
2. Facilitate Learning Experiences
3. Classroom-Based Assessment
4. SLAR

Learning Experience	
Learning Outcome	N.3 b. Investigate situations involving proportionality so that they can solve problems involving average speed, distance and time.
Topic	Distance, Speed, Time
Investigation	How fast can humans run?

Problem-Solving Cycle



Define a Problem

What is the problem?	Why does it interest you?	How many answers are you expecting?	What elements are you trying to focus on?	What steps will you follow?

Translate to Maths

What ideas have you thought about using so far?	Why have you decided not to use some strategies?	What units/ quantities are relevant?	Why do you think other strategies will be more effective?

Engage & Solve

Where did you get those numbers?	What pictures, graphs or diagrams might help represent your ideas?	What mathematical ideas did you use? Why did you use them?

Interpret

How do you know your calculations are correct? Do they always work?	When might it not work and why?	What could you have done differently?	Has your solution generated more problems? What kind?

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Junior CYCLE for teachers

Features of Quality – Mathematical Investigation

Features of Quality are the criteria used to assess the level of student achievement in a Classroom-Based Assessment (CBA). Described below are the Features of Quality for the Mathematical Investigation.

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	Yet to Meet Expectations	In Line with Expectations	Above Expectations	Exceptional
Defining the Problem Statement	Uses a given problem statement and with guidance breaks the problem down into steps	With guidance poses a problem statement, breaks the problem down into manageable steps and simplifies the problem by making assumptions, if appropriate	With limited guidance poses a problem statement and clarifies/simplifies the problem by making reasonable assumptions, where appropriate	Poses a concise problem statement and clarifies and simplifies the problem by making justified assumptions, where appropriate
Finding a Strategy or Translating the Problem to Mathematics	Uses a given strategy	Chooses an appropriate strategy to engage with the problem	Justifies the use of a suitable strategy to engage with the problem and identifies any relevant variables	Develops an efficient justified strategy and evaluates progress towards a solution where appropriate; conjectures relationship between variables where appropriate
Engaging with the Mathematics to Solve the Problem	Records some observations/data and follows some basic mathematical procedures	Records observations/data and follows suitable mathematical procedures with minor errors; graphs and/or diagrams/ words are used to provide insights into the problem and/or solution	Records observations/data systematically, suitable mathematical procedures are followed, and accurate mathematical language, symbolic notation and visual representations are used; attempts are made to generalise any observed patterns in the solution/observation	Mathematical procedures are followed with a high level of precision, and a justified answer is achieved; solution/observations are generalised and extended to other situations where appropriate
Interpreting and Reporting	Comments on any solution	Comments on the reasonableness of the solution where appropriate and makes a concrete connection to the original question, uses everyday familiar language to communicate ideas	Checks reasonableness of solution and revisits assumptions and /or strategy to iterate the process, if necessary, uses formal mathematical language to communicate ideas and identifies what worked well and what could be improved	Deductive arguments used and precise mathematical language and symbolic notation used to consolidate mathematical thinking and justify decisions and solutions; strengths and/ or weaknesses in the mathematical representation/ solution strategy are identified

Source: *Junior Cycle Mathematics Guidelines for the Classroom-Based Assessments and Assessment Task*, November 2019.

During the CBA and SLAR meeting, teachers should refer to the most recent publication of the Assessment Guidelines available at www.curriculumonline.ie.

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Junior CYCLE for teachers

Features of Quality – Mathematical Investigation

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How can you simplify the problem?
What assumptions do you need to make?
Is there any potential problems?

	Yet to Meet Expectations	In Line with Expectations	Above Expectations	Exceptional
Defining the Problem Statement			With limited guidance poses a problem statement and ... where appropriate	Poses a concise problem statement and clarifies and simplifies the problem by making justified assumptions, where appropriate
Finding a Strategy or Translating the Problem to Mathematics	Uses a given strategy	Chooses an appropriate strategy to engage with the problem	Justifies the use of a suitable strategy to engage with the problem and identifies any relevant variables	Develops an efficient justified strategy and evaluates progress towards a solution where appropriate; conjectures relationship between variables where appropriate
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Finding a Strategy or Translating the Problem to Mathematics			ustifies the use of a suitable strategy to engage with the problem and	Develops an efficient justified strategy and evaluates progress towards a solution where appropriate; conjectures relationship between variables where appropriate
Engaging with the Mathematics to Solve the Problem	Records some observations/data and follows some basic mathematical procedures	Records observations/data and follows suitable mathematical procedures with minor errors; graphs and/or diagrams/ words are used to provide insights into the problem and/or solution	Records observations/data systematically, suitable mathematical procedures are followed, and accurate mathematical language, symbolic notation and visual representations are used; attempts are made to generalise any observed patterns in the solution/observation	Mathematical procedures are followed with a high level of precision, and a justified answer is achieved; solution/observations are generalised and extended to other situations where appropriate
Interpreting and Reporting	Comments on any solution	Comments on the reasonableness of the solution where appropriate and makes a concrete connection to the original question, uses everyday familiar language to communicate ideas	Checks reasonableness of solution and revisits assumptions and /or strategy to iterate the process, if necessary, uses formal mathematical language to communicate ideas and identifies what worked well and what could be improved	Deductive arguments used and precise mathematical language and symbolic notation used to consolidate mathematical thinking and justify decisions and solutions; strengths and/ or weaknesses in the mathematical representation/ solution strategy are identified

How can you simplify the problem?
What assumptions do you need to make?
Is there any potential problems?

Which strategy would be most effective?
What have you decided not to use, why?
What units/quantities are you using?

Source: *Junior Cycle Mathematics Guidelines for the Classroom-Based Assessments and Assessment Task*, November 2019.

During the CBA and SLAR meeting, teachers should refer to the most recent publication of the Assessment Guidelines available at www.curriculumonline.ie.

An tSraith Shóisearach do Mhúinteoirí
JuniorCYCLE for teachers

Features of Quality – Mathematical Investigation

Features of Quality are the criteria used to assess the level of student achievement in a Classroom-Based Assessment (CBA). Described below are the Features of Quality for the Mathematical Investigation.

	Yet to Meet Expectations	In Line with Expectations	Above Expectations	Exceptional
Defining the Problem Statement	How can you simplify the problem? What assumptions do you need to make? Is there any potential problems?		With limited guidance poses a problem statement and … where appropriate	Poses a concise problem statement and clarifies and simplifies the problem by making justified assumptions, where appropriate
Finding a Strategy or Translating the Problem to Mathematics	Which strategy would be most effective? What have you decided not to use, why? What units/quantities are you using?		…ustifies the use of a suitable strategy to engage with the problem and …	Develops an efficient justified strategy and evaluates progress towards a solution where appropriate; conjectures relationship between variables where appropriate
Engaging with the Mathematics to Solve the Problem	How accurate are our solutions? Are the solutions always correct? What other situations might this work or not work?		Records observations/data systematically, suitable mathematical procedures are followed and … representations are used; attempts are made to generalise any observed patterns in the solution/observation	Mathematical procedures are followed with a high level of precision, and a justified answer is achieved; solution/observations are generalised and extended to other situations where appropriate
Interpreting and Reporting	Comments on any solution	Comments on the reasonableness of the solution where appropriate and makes a concrete connection to the original question, uses everyday familiar language to communicate ideas	Checks reasonableness of solution and revisits assumptions and /or strategy to iterate the process, if necessary, uses formal mathematical language to communicate ideas and identifies what worked well and what could be improved	Deductive arguments used and precise mathematical language and symbolic notation used to consolidate mathematical thinking and justify decisions and solutions; strengths and/ or weaknesses in the mathematical representation/ solution strategy are identified

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Interpreting and Reporting	How do you know your work is correct? What situations does it work/not work? What other problems/ issues have your solution raised?		Checks reasonableness of solution and revisits assumptions and /or strategy to iterate the process, if ... dentifies what worked we... what could be improved	Deductive arguments used and precise mathematical language and symbolic notation used to consolidate mathematical thinking and justify decisions and solutions; strengths and/ or weaknesses in the mathematical representation/ solution strategy are identified

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Mathematical Investigation	Guiding Questions	Student Activity
Problem	• What do we need to consider? • Why is this problem relevant? • Who does it concern? • What can we compare humans with?	• Research speeds of top sprinters, animals, etc. • Convert all to similar units
Translate to Mathematics	• What are the variables? What is changing? • What are the constants? What can we keep the same? • What assumptions do we need to make? • What techniques might we use? • What other areas of learning can we link this to?	• Discussion on what to keep constant (distances), and what is changing. • Discussion on why assumptions are important (same terrain, etc). • Link to graphing functions and functions.
Engage & Solve	• What experiment can we set up? • What information can we collect? • What other things do we need to consider? • Can we represent our findings using graphs, tables, diagrams?	• Bring outside, measure distance, record times and find speed. • Convert to similar units m/s, km/h. • Graphing results using a linear graph. • Could include discussion on average speed and not instantaneous speed. • Create a table/ graph to represent results. Average speed of teenager, adult, sprinter. How does that compare to an animal.
Interpret	• How accurate is our answer? • Can we extend our work in anyway to make it better? • What situation may our answer not work in? • How does our work compare to what it tells us online? • What limitations are there to our research?	• Limitations that might have existed – terrain, gear, etc. • Discussion on differences in speed between humans (teenagers and sprinters) and animals.

Learning Experiences

- There are no “right” answers.
- Problems can be extended as the students see fit, allowing them to show off various extensions to the task.
- Problems allow students to look at a problem from a number of different perspectives and build a number of different solutions.
- Allows a chance for argumentation and discussion to develop.

Scaffolding CBA's Questioning

Preparation

Scaffolding CBA’s Questioning

Preparation

Use Features of Quality

Scaffolding CBA’s Questioning

Preparation

Use Features of Quality

Wait Time

Scaffolding CBA's Questioning

Preparation

Use Features of Quality

Wait Time

Student Questions

Scaffolding CBA's Questioning

- Preparation
- Use Features of Quality
- Wait Time
- Student Questions
- Groupwork Activities

Reflection:

- How did you use Learning Experiences or Problem Solving before?
- How might your practice change as a result of this video?
- Do you see these types of classes as beneficial to your students?

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