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Topic	**Trigonometry**
Year Group	Junior Cycle 2nd / 3rd Years
Learning Intentions	• GT.1 (a) Investigate 2D shapes so that they can draw and interpret scaled diagrams. • GT4: Evaluate and use trigonometric ratios (sin/cos/tan) and their inverses, involving angles between 0° and 90° at integer values and in decimal form. • GT.3 (b – ii) Recall and use the concepts of theorem 14 (In a right-angled triangle, the square of the hypotenuse is the sum of the squares of the other two sides) and appropriate converses including relevant operations involving square roots. • N2a: Use and understand ratio and proportion. • AF7: Represent and interpret functions in different ways.

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	Lessons	Teacher CPD Focus
1	Triangles & Right-Angle Exploration	
2	Pythagoras Theorem (Recap)	Pythagoras' Theorem – Common Misconceptions/ Mistakes
3	Relationship between Angles & Sides	Transforming Textbook Questions → Authentic Problems
4	Introduction to Sin/Cos/ Tan → Functions	Sin/Cos/Tan → Unit Circle (Measurements)
5	Sin/ Cos/ Tan → Ratio of Sides	
6	Solving Trigonometric Equations → Finding Sides	
7	Inverse Trigonometric Functions → Finding Angles	Linking Trig & Inverse Trig Functions
8	Learning Experience: Real-Life Application	Supporting CBAs through the Problem-Solving Cycle & Questioning

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	Lessons	Resources
1	Triangles & Right-Angled Triangles Exploration	Shape Exploration
2	Pythagoras Theorem (Recap)	Alpine Skiing Activity
3	Right Angled Triangle – Relationship between Sides & Angles	Toothpick/Ladder Activity
4	Introduction to Sin/Cos/Tas as a Function	Graphing Functions (Junior Cycle Sample Paper)
5	Sin/Cos/Tan → Ratio of Sides	Ratio of Sides Worksheet
6	Solving Trigonometric Equations → To Find Side	Tennis Activity – Solving Trigonometric Equations
7.	Inverse Trigonometric Functions ➔ To Find Angle	High Heel Activity → Finding angles
8	Learning Experience: Real-Life Problem (Exploration of a Problem – No definite answer)	Ramp Activity → Scaffolded with Problem Solving Cycle

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Lesson	Triangles & Right-Angle Exploration
Learning Intentions	**GT.1 (a) Investigate 2D shapes so that they can draw and interpret scaled diagrams.**

Teacher Notes

• Trigonometry is the study of triangles – their shape and the relationship between their angles and sides.
• **Individual Activity – Shape Exploration**: Students try and create as many triangles as they can using a variety of shapes and objects (shape exploration task).
• **Class Discussion** – Students comment on the process of identifying shapes and what they noticed when completing the activity.
• **Groupwork Activity**: Get the students in small groups to identify as many triangles as they can inside or outside the classroom. Students could take photos using school devices and make an album of real-life triangles.
• **Class Discussion**: Right-Angled Triangles
  ○ What are they?
  ○ What do they look like?
  ○ Are they equilateral, isosceles or scalene?
  ○ Why might we want to use right-angle triangles?
  ○ How do we know a triangle is right angled?
  ○ Why might right angle triangles be useful for us to identify?
• Revisit **Individual Activity #1 and/or Groupwork Activity**: How many right-angle triangles had we identified originally?

Accompanying Resources
• **Individual Activity – Shape Exploration**

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Lesson	Pythagoras Theorem (Recap)
Learning Intentions	**GT.3 (b - ii) Recall and use the concepts of theorem 14 and appropriate converses including relevant operations involving square roots.**

Teacher Notes

• Pythagoras' Theorem is usually covered with Area & Volume, to find vertical heights and sides missing. This will act as a recap of Pythagoras' Theorem.
• **Recap Pythagoras Theorem** referring to the theorem and converse.
• **Class Discussion**: Theorem & Converse
  ○ Why do we need theorems?
  ○ Why is Pythagoras' Theorem useful for us?
  ○ When would we use the converse of the theorem?
  ○ Does every theorem have a converse, why?
• **Student Activity**: Ask students to come up with an example of a question that uses Pythagoras' Theorem and a second question which focuses on the Converse of Pythagoras' Theorem. Students can contribute questions to the class for the whole class to try and answer.
  ○ Theorem: Find the value of x, in the triangle…
  ○ Converse: Do the sides 5, 12, 13 make up a right-angled triangle?
• **Pairwork Activity** – Alpine Skiing: Students will explore the use of Pythagoras theorem in finding missing sides in a real-life example.

Accompanying Resources
• **Pairwork Activity – Alpine Skiing**

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Lesson	Relationship between Angles & Sides
Learning Intentions	**GT.4 evaluate and use trigonometric ratios and their inverses, involving angles between 0° and 90° at integer values and in decimal form**

Teacher Notes

• **Class Discussion**: Angles & Sides of Triangles
  ○ What types of triangles are there?
  ○ What type of angles do we have in a triangle?
  ○ Could there be two right angles in a triangle? Why?
  ○ What is the name given to shapes that have the exact same angles?
  ○ What about isosceles, equilateral and scalene triangles?
• **Pairwork Activity**: Students draw right-angled triangles with obtuse angles and acute angles.
  ○ Do we notice any relationship between the angles and sides?
  ○ Where is the biggest side located? Where is the smallest side located?
• **Individual Activity – Toothpick Activity:** Ask the students to use the toothpick to create different angles with the wall. Class exploration measuring different angles and sizes using a protractor and a ruler.
• **Class Discussion:** Exploration of relationship between sides and angles.
  ○ What did you notice about the different angles in a triangle?
  ▪ Size of the angle changes the lengths of the sizes
  ▪ There's a relationship between the angles and the sides
  ▪ The two angles (A and B) can only add up to 90.
  ○ Where is the longest side in a right-angle triangle?
  ○ Where is the smallest side?
  ○ What conclusion can we make about sides and angles in triangles?

Accompanying Resources
Individual Activity – Toothpick Activity

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Lesson	Introduction to Sin/Cos/ Tan → Functions
Learning Intentions	**AF.7 Investigate functions so that they can:** **a. demonstrates understanding of the concept of a function** **b. represents and interpret functions in different ways — graphically, diagrammatically, in words, and algebraically**

Teacher Notes

• **Functions – Recap**
  ○ Inputs and Outputs (Domain/Range)
  ○ Graphing Functions → Graph a linear/quadratic functions
  ○ What makes a function a function? Every input has one, and only one output.
• **Introduction:** Sin/Cos/ Tan as functions.
  ○ y = sinx/ y = cosx / y = tanx
  ○ x = Input (Angle in Degrees)
  ○ y = Output (Ratio of the Sides)
• **Individual Activity:** Graphing Cos & Sin (Junior Cycle Sample Paper)
• **Class Discussion:** Differences and similarities between sin and cos functions, including what are the inputs and what are the outputs.
• **Conclusion:** Students should have gained an understanding about sin/cos/tan as functions of an angle, and how that might relate to a ratio (decimal)

Accompanying Resources
Individual Activity: Graphing Cos & Sin

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Lesson	Sin/ Cos/ Tan → Ratio of Sides
Learning Intentions	**GT.4 Evaluate and use trigonometric ratios and their inverses, involving angles between 0° and 90° at integer values and in decimal form**

Teacher Notes

• **Recap**: Sin/Cos/Tan as Functions
  ○ Input = x-value = Degrees
  ○ Output = y-value = Ratio of the Sides
• **Introduction**: SOHCAHTOA → Trigonometric Ratios
  ○ Sin (Angle) equates to the opposite side divided by the hypotenuse.
  ○ Cos (Angle) equates to the adjacent side divided by the hypotenuse.
  ○ Tan (Angle) equates to the opposite side divided by the adjacent
• **Instruction**: Labelling a right-angled triangle → All depends on where the (Angle) you are focusing on is. This is the input to the function
  ○ Hypotenuse is always across from the right-angle (and the longest side).
  ○ Opposite is directly across from the (Angle) you are focusing on.
  ○ Adjacent is beside the (Angle) you are focusing on.
• Class Activity: Display different right-angled triangles on the board and ask students to work together to label them, based on a given angle.
• **Individual Activity: Exploration of Trigonometric Ratios**
• **Pairwork**: Discussion of Problems & Summarising of Rules Learned
  ○ What did you notice from the table you completed?
  ○ Can we make any rules that would summarise what we have learned?
  ○ Why might these rules be useful?

Accompanying Resources
Individual Activity: Exploration of Trigonometric Ratios

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Lesson	Solving Trigonometric Equations → Finding Sides of Triangles
Learning Intentions	**GT.4 evaluate and use trigonometric ratios and their inverses, involving angles between 0° and 90° at integer values and in decimal form** **AF.4 select and use suitable strategies (graphic, numeric, algebraic, trial and improvement, working backwards) for finding solutions to a. linear equations in one variable with coefficients in Q and solutions in Z or in Q**

Teacher Notes

• **Recap:** Discussion on learning so far, including setting up trigonometric ratios SinA = O/H, CosA = A/H, TanA = O/A
• **Introduction**: Recap on solving linear equations with fractions, including discussions on eliminating the denominators and the inverse operations that are used in linear equations.
• **Class Discussion**: Introduce students to two linear and two trigonometric equations, with a side missing in numerator and denominator (Class Activity Resource Sheet)
  ○ Discussion surrounding similarities and differences between linear equations and trigonometric equations, including location of variable.
  ○ Students work on solving the linear equations in groups. Class discusses solutions and how the answer was found.
  ○ Students work on solving the trigonometric problems, based on discussion.
  ○ Solutions are talked through as a class.
• **Student Activity**: Students work on activity to create trigonometric equations, using Sin/Cos/Tan, and solve them using the methods discussed in the lesson. This will be a tennis-activity to allow students to see the real-life application of this topic

Accompanying Resources
Class Activity: 2 x Linear and 2 x Trigonometric Equations **Individual Activity: Tennis Activity – Creating & Solving Trigonometric Equations**

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Lesson	Inverse Trig Functions → Finding Angles
Learning Intentions	**GT.4 evaluate and use trigonometric ratios and their inverses, involving angles between 0° and 90° at integer values and in decimal form**

Teacher Notes

• **Recap:** Discussion on learning that has taken place so far.
  ○ Identifying right angled triangles.
  ○ How to correctly label the hypotenuse, adjacent and opposite sides.
  ○ What Sin/Cos/Tan can be used for.
• **Introduction:** Learning to find an Angle
  ○ Like all functions, we can be given an input, and use that to help us find an output.
  ○ We can also use our output to find a particular input.
  ○ Can use example of going from x squared to x.
• **Class Example:** Teacher goes through an example of a right angle triangle, with two known sides and angle missing. Teacher probes to develop understanding.
  ○ Why is this question different to what we have done before?
  ○ Can we still form a Sin/Cos/Tan equation?
  ○ Which sides/ ratio should we use?
• **Instruction:** When a trigonometric equation is formed, teacher instruction on how to solve the equation.
  ○ To isolate the angle, we need to remove the sin/cos/tan part from the angle.
  ○ To do this, we use Sin/Cos/Tan inverse.
  ○ Teacher shows example on the board and shows students button on the calculator.
  ○ Teacher and students do another 2 examples together**.**
• **High Heel Activity –** Students work on a worksheet based on the angles in high heels to apply the knowledge that they've learned in this class.

Accompanying Resources
Individual Activity: High Heel Activity

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Lesson	Learning Experience: Real-Life Application
Learning Intentions	**GT.4 evaluate and use trigonometric ratios and their inverses, involving angles between 0° and 90° at integer values and in decimal form**

Teacher Notes
This class will be for students to work on a learning experience, that will summarise their learning so far in trigonometry. The teacher should facilitate and support the lesson and encourage students to push the boundaries and be creative with the project, reminding them that the mathematical element is the most important.

Accompanying Resources
Ramp Activity – Problem Solving Cycle

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